Expert Systems With Applications 288 (2025) 128196

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Multi-attribute decision-making approach for improvisational emergency supplier selection: Partial ordinal priority approach

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ARTICLE INFO

Keywords:
Improvisational emergency supplier selection
Multi-attribute decision-making
Partial-order relationship
Adversarial hasse diagram
Partial ordinal priority approach

ABSTRACT

The unpredictable and complex nature of disasters underscores the need for a scientific and logical approach to improvisational emergency supplier selection (IESS), which is a typical multi-attribute decision-making problem. This study introduces Partial Ordinal Priority Approach (OPA-P) for IESS under information uncertainty and multi-stakeholder involvement. OPA-P contributes to a novel partial-order extension of Ordinal Priority Approach, emphasizing the necessity of Pareto-optimal analysis. It consists of two main steps: the first stage optimizes decision weights through a linear programming problem using easily accessible and stable ordinal preference information to simultaneously determine the weights of experts, attributes, and alternatives; the second stage generates the adversarial Hasse diagram for alternative comparison derived from the partial-order cumulative transformation set. This diagram streamlines the redundant dominance structure among alternatives and provides information on Pareto-optimal and suboptimal alternatives along with their clusters. The proposed approach is demonstrated through a case study on IESS for the Zhengzhou mega-rainstorm disaster with sensitivity and comparison analysis for model validation. Overall, the novelty of OPA-P lies in its ability to facilitate swift and stable decision-making while identifying potential Pareto-optimal solutions amidst high information uncertainty and multi-stakeholder involvement.

1. Introduction

In today's high-risk society, disasters such as earthquakes, floods, terrorist attacks, and pandemics pose significant threats to human life, property, and societal stability (Wang, Wang, Li, & Li, 2022). These events often exhibit suddenness, uncertainty, and complexity, complicating efforts to predict and control their impacts (Song, Tappia, Song, Shi, & Cheng, 2024). The aftermath of such disasters frequently involves widespread infrastructure damage, transportation disruptions, communication breakdowns, and shortages of emergency supplies, leading to substantial losses in both lives and property (Yan, Hu, Wang, & Xia, 2025). A notable example is the COVID-19 outbreak in Wuhan, Hubei province, where the rapid spread of the virus overwhelmed the medical system, causing a severe shortage of medical resources. Hospitals struggled to handle the surge in infected patients, while frontline workers faced the risk of infection due to shortages in personal protective equipment. This scarcity forced medical personnel to reuse protective gear, which further exacerbated the risk of viral transmission. In response, the Chinese government allocated millions of medical masks, protective suits, and thermometers to Hubei to address these urgent needs (People'sDaily, 2020). Therefore, during disaster response, the efficiency and resilience of the supply chain are pivotal. Ensuring the timely delivery of supplies to affected areas can significantly reduce response times, enhance rescue operations, and mitigate disaster-related losses (Jiang, Liu, Wang, Ding, & Zhang, 2024). As such, selecting appropriate emergency suppliers is crucial. This process, a type of multi-attribute decision-making (MADM), involves evaluating and selecting suppliers based on multiple incomparable factors while considering the diverse interests of various stakeholders (Li, Sun, & Fei, 2025; Liu, Tu, Zhang, & Shen, 2024; Zhang, Wei, & Chen, 2022). However, in the context of extreme disasters, traditional supplier selection methods may fall short due to the need for rapid decision-making in an environment of high uncertainty, where existing pre-disaster supplier arrangements often fail to meet immediate demands. This highlights the importance of improvisational emergency supplier selection (IESS)–a critical and evolving process in disaster management. Unlike conventional MADM

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https://doi.org/10.1016/j.eswa.2025.128196
Received 3 September 2023; Received in revised form 26 April 2025; Accepted 15 May 2025
Available online 22 May 2025

problems, IESS must address the challenges of rapid, informed decision-making amidst high uncertainty, often with the involvement of multiple stakeholders (Li, Kou, & Peng, 2022a; Su, Zhao, Wei, & Chen, 2022).

Through the literature review, three critical limitations in current research have been identified: (1) Most existing MADM methods often lack stability and fail to effectively analyze potential Pareto-optimal solutions. This limits the transparency and stability of the decision-making process, which could be enhanced by incorporating Pareto-optimal solutions to provide more comprehensive alternatives (Fang, Zhou, & Xiong, 2024; Wu, Lu, Li, & Deng, 2023). (2) Existing studies heavily rely on expert opinions, which are easier to obtain but require substantial professional knowledge for evaluating values, semantic judgments, and pairwise comparisons (Afrasiabi, Tavana, & Caprio, 2022). This process is cumbersome and time-consuming, especially when dealing with multiple stakeholders, leading to significant variations in decision data that can introduce errors and disconnects between decision outcomes and actual circumstances. (3) Most existing MADM methods in IESS often involve data standardization, expert opinion aggregation, and the pre-acquisition of weight information, which increases the complexity of the model. Additionally, current aggregation methods based on algebraic logic often fail to objectively reflect the actual preferences and viewpoints of experts, potentially leading to decision outcomes that do not align with real-world conditions (Ataei, Mahmoudi, Feylizadeh, & Li, 2020).

To overcome the above limitations in IESS, this study proposes the Partial Ordinal Priority Approach (OPA-P) for solving the IESS problem. Specifically, the proposed approach is based on the Ordinal Priority Approach (OPA) with the ranking data of the experts, attributes, and alternatives as model inputs. Based on the ordinal preference, this study derives a decision weight optimization model and partial-order cumulative transformation to generate the most simplified dominance structure of alternatives (adversarial Hasse diagram). The proposed approach can simultaneously determine the weight of experts, attributes, and alternatives and generate dominance structure with information on Pareto-optimal alternatives, sub-optimal alternatives, and clustering details. The primary contribution of this study lies in proposing OPA-P for IESS under information uncertainty and multi-stakeholder involvement while consider the Pareto-optimal analysis. Specifically:

- **Methodology.** This study incorporates the partial-order theory into the original OPA model for the first time, achieving a partial-order extension of OPA. The partial-order cumulative transformation and adversarial Hasse diagram generation of OPA-P can effectively identify potential Pareto-optimal solutions and facilitate more robust decision-making.
- **Theory.** This study demonstrates the theoretical foundation of the partial-order set derived from partial-order cumulative transformation by deriving the order-preserving properties and the its relationships with the total order set based on a single comprehensive evaluation value and the partial order set based on strict Pareto-optimality. This proof provides a theoretical basis for not only OPA-P but also other MADM methods in generating dominance structures and identifying Pareto-optimal alternatives.
- **Practice.** This study utilizes OPA-P by integrating a representative attribute system for IESS, offering insights and guidance for authentic decision-making processes. Furthermore, OPA-P is not limited to IESS but is also applicable to any decision scenario exhibiting similar characteristics to IESS.

The remaining parts of this paper are organized as follows: Section 2 conducts a literature review of the MADM approach in IESS. Section 3 outlines the evaluation attributes for IESS. In Section 4, the research method (OPA-P) is presented. Section 5 employs the IESS of Zhengzhou mega-rainstorm disaster as the case study to demonstrate and validate OPA-P. Lastly, Section 6 presents the conclusions and outlines future directions.

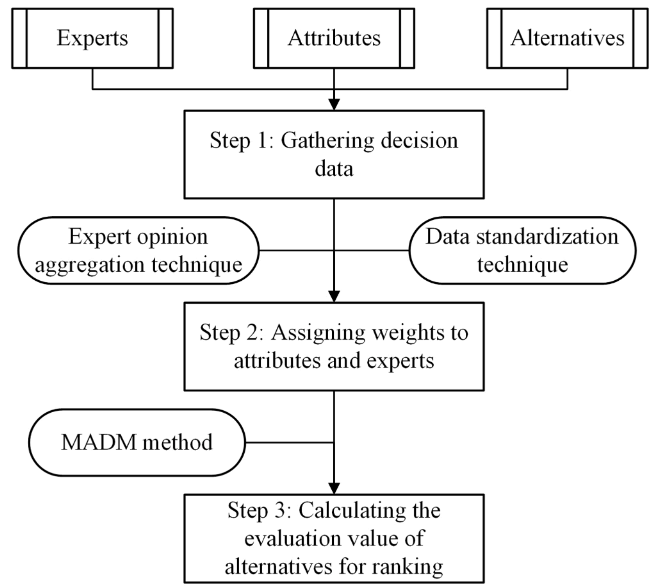


Fig. 1. Implementation steps of the MADM for IESS.

2. Literature review

When confronted with emergencies, organizations and governments require swift access to emergency supplies to cater to people's fundamental needs and facilitate disaster relief operations (Zhang et al., 2022). The process of selecting emergency suppliers assumes a pivotal role, as their product quality, service responsiveness, and supply chain reliability directly impact the efficacy of rescue operations and the well-being of those affected (Wang et al., 2022). The selection of emergency suppliers can be conceptualized as a classical MADM problem (Li et al., 2025; Liu et al., 2024; Zhang et al., 2022). In current research concerning emergency supplier selection, scholars typically execute it in three steps, as illustrated in Fig. 1 (Liu, He, Chan, & Wang, 2022). The initial step involves gathering decision-making data concerning evaluation attributes, experts, and alternatives, encompassing historical records, statistical data, or expert opinions. The subsequent step entails determining the weights assigned to attributes and experts. Ultimately, rational MADM methods are utilized to ascertain the comprehensive evaluation value for ranking alternatives. However, it is noteworthy that the selection of emergency suppliers mainly focuses on the emergency preparedness phase, and the decision-making process is typically unrestricted by severe time pressures (Li, Yang, & Xiang, 2022b; Liu et al., 2022). Nevertheless, disaster events often entail complexity and uncertainty, leading to the possibility that pre-selected emergency suppliers may not adequately meet the requirements for disaster response (Pamucar, Torkayesh, & Biswas, 2022). In such instances, improvisational emergency supplier selection (IESS) becomes essential, exhibiting characteristics distinct from conventional emergency supplier selection. Specifically, IESS must efficiently acquire and aggregate expert opinions, considering incomplete preferences information, to achieve stable decision outcomes in the situations characterized by intense time pressure, poor quality or difficult accessibility of decision data, and the involvement of multiple stakeholders.

In the present IESS research, widely utilized MADM methods encompass prominent techniques such as GRA (Zhang et al., 2022), TOPSIS (Afrasiabi et al., 2022; Ge, Yang, Wang, & Shao, 2020), VIKOR (Zhang, Zheng, Tian, & Wei, 2023; Zhu & Wang, 2023), TODIM (Liu et al., 2024; Wang, Liang, Li, & Luo, 2023), BWM (Song et al., 2024), MARCOS (Rong & Yu, 2024), MABAC (Dai, Li, Zhou, & Wu, 2024), MACBETH (Pamucar et al., 2022), DEMATEL (Wu & Liao, 2024), and others. Considering the challenges associated with acquiring data in emergency situations, only a few studies incorporate objective decision data, while the majority rely on subjective decision data derived from expert opinions due to poor data quality. Subjective decision data takes the form of evaluation values (Ge et al., 2020), semantic values (Li et al., 2022b; Su et al., 2022; Wang et al., 2023), pairwise comparison values (Liu et al., 2022; Wang et al., 2023), and hybrid subjective data (Wang, 2024c). Given the uncertainty, ambiguity of crises, and limitations in expert experience and available information, the subjective decision data provided by experts also carries a level of uncertainty and ambiguity. Consequently, some studies apply grey system theory (Wang, 2024b; Zhang et al., 2022), fuzzy theory (Ding, Liang, Cheng, & Ji, 2025; Liu, Pan, Zhu, & Wu, 2023; Pamucar et al., 2022), and rough set theory (Afrasiabi et al., 2022; Pamucar et al., 2022; Rong & Yu, 2024) to enhance subjective decision data, addressing inherent uncertainties and ambiguities. Regarding expert opinion aggregation, most studies mainly rely on algebraic logic-based aggregation techniques, such as weighted averages and geometric means (Ataei et al., 2020; Pamucar et al., 2022). Only a few studies explore the application of expert consensus and trust networks in the aggregation of expert opinions in IESS (Liu et al., 2024). Additionally, to account for risk preferences of decision-makers arising in emergency scenarios, numerous current studies integrate prospect theory (Zhang et al., 2023), cumulative prospect theory (Liao, Qin, Wu, Yazdani, & Zavadskas, 2020; Zhang et al., 2022), and regret theory (Liu et al., 2023) into the IESS decision-making process. For example, Ding et al. (2025) proposes a new emergency supplier evaluation framework that uses probabilistic hesitant fuzzy sets as input data to address the problem of expert weight allocation in large-group decision-making by introducing a two-layer weight model and incorporat a clustering-based method for ranking decision alternatives. Dai et al. (2024) proposes a healthcare supplier selection approach using belief distributions as input data and combines SMAA and MABAC methods to address uncertainties and incomplete information in decision-making, providing accurate rankings of suppliers. Liu et al. (2024) presents a framework for selecting emergency material suppliers by using expert evaluations in the form of hesitant fuzzy linguistic sets and quantitative data, employing the HF-ExpTODIM method and game theory to address group decision-making challenges under emergency conditions. Su et al. (2022) utilized probabilistic linguistic values provided by experts as input data and employed a TODIM approach, augmented with prospect theory, for assessing suppliers, tackling the complexities of MADM in the intricate and dynamic environment.

The essence of the above MADM process lies in projecting the supplier's performance or utility evaluated by experts on various attributes into a single comprehensive evaluation attribute for supplier ranking. However, results obtained through the above approach often lack stability and fail to analyze potential Pareto-optimal solutions in IESS (Fang et al., 2024). The practice has shown that considering Pareto-optimal solutions in decision analysis can effectively enhance the transparency and stability of decision-making while also providing insights for decision portfolios. In addition, despite the prevailing tendency in most studies to rely on expert opinions, which are more readily available compared to objective decision data, the evaluation values, semantic values, and pairwise comparison values also require the support of substantial professional knowledge and information (Afrasiabi et al., 2022). Experts need to provide specific estimates for alternatives, but this process can be cumbersome and time-consuming, especially when obtaining pairwise comparison values. It also needs to incorporate a range of methods involving data standardization, expert opinion aggregation, and pre-acquisition of weight information, which adds complexity to the model and increases the likelihood of errors. Moreover, the involvement of multiple stakeholders results in significant variations in decision data. However, the current expert opinion aggregation methods based on algebraic logic often struggle to objectively reflect the actual views and preferences of experts in the above context (Ataei et al., 2020). Such disparities may lead to a disconnect between decision outcomes and actual circumstances.

Ordinal preferences (i.e., ranking data) are generally more accessible for experts to elicit and offer greater stability and reliability for decision-makers (Wang, Peng, & Kou, 2021), as they require only comparative judgments without quantifying differences among alternatives. Thus, utilizing ranking data in IESS represents a promising direction. Among various multi-attribute decision-making (MADM) approaches, the Ordinal Priority Approach (OPA), proposed by Ataei et al. (2020), is an optimization-based method grounded in ordinal preferences and has demonstrated potential for solving IESS problems. OPA determines the weights of experts, attributes, and alternatives simultaneously by solving a linear programming model, eliminating the need for data normalization and expert opinion aggregation. Due to its practical applicability, OPA has gained considerable attention in recent years. Recent developments include fuzzy OPA (Zhao, Hendalianpour, & Liu, 2024), rough set OPA (Zhao et al., 2024), grey OPA (Mahmoudi, Deng, Javed, & Zhang, 2021b), and robust OPA (Mahmoudi, Abbasi, & Deng, 2022b; Wang, 2024c) for handling data uncertainty; TOPSIS-OPA (Mahmoudi, Deng, Javed, & Yuan, 2021a), DEMATEL-OPA (Zhao et al., 2024), and DGRA-OPA (Wang, 2024b) for large-scale group decision-making; and DEA-OPA (Cui & Wang, 2024; Cui, Wang, Li, & Bai, 2025; Mahmoudi, Abbasi, & Deng, 2022a) for relative efficiency analysis. However, no study has yet examined the extension of OPA under partial order theory, which could offer new insights into Pareto analysis in decision-making problems.

Hence, this research endeavors to present a novel MADM method based on OPA for IESS that considers information uncertainty, multi-stakeholder involvement, and Pareto-optimal analysis. It leverages more reliable and easily accessible ranking data as input, eliminating the necessity for data standardization, expert opinion aggregation, and pre-weight acquisition techniques.

3. Evaluation attributes of IESS

This section presents the evaluation attributes in alignment with the objectives of IESS obtained from literature, focusing on supplier emergency response capability and emergency supply capacity, as outlined in Table 1.

3.1. Supplier emergency response capability

Disasters cause instability, urgency, and complexity, which can disrupt supply chains, delay logistics, and lead to inventory shortages, highlighting the critical role of suppliers' emergency response capabilities. Emergency response capability refers to a supplier's ability to swiftly adapt supply chains to maintain continuous production and delivery during crises (Pamucar et al., 2022). Based on literature review, five core attributes are identified to assess emergency response capability: response speed (C1), delivery reliability (C2), geographic coverage (C3), sustainability (C4), and collaborative experience and credibility (C5).

- **Response speed (C1)**: Response speed refers to the supplier's ability to quickly implement emergency measures, ensuring prompt response and minimizing delays (Li et al., 2022a; Pamucar, Yazdani, Obradovic, Kumar, & Torres-Jiménez, 2020).

Table 1
Evaluation attributes for IESS.

Perspective	Attribute
Supplier emergency response capability	Response speed (C1)
	Delivery reliability (C2)
	Geographic coverage (C3)
	Operation sustainability (C4)
	Collaborative experience and credibility (C5)
Emergency supply capacity	Availability (C6)
	Quality (C7)
	Cost-effectiveness (C8)

- **Delivery reliability (C2)**: Delivery reliability represents the supplier's ability to reliably delivery emergency supplies during critical times, with high reliability ensuring timely and precise distribution (Afrasiabi et al., 2022; Pamucar et al., 2020).
- **Geographic coverage (C3)**: Geographic coverage depicts the extent to which the supplier can deliver emergency supplies to affected areas, with broad coverage enabling rapid support in disaster-stricken regions (Li et al., 2022a; Pamucar et al., 2022).
- **Sustainability (C4)**: Sustainability denotes the supplier's ability to maintain reliable service during crises while integrating sustainable practices across economic, social, and environmental dimensions (Kannan, Mina, Nosrati-Abarghooee, & Khosrojerdi, 2020).
- **Collaborative experience and credibility (C5)**: Collaborative experience and credibility reveal the supplier's proven track record and reliability in previous emergency collaborations, with strong experience enabling better coordination and integration with various stakeholders during disaster responses (Li et al., 2022a; Pamucar et al., 2022).

These attributes ensure rapid, reliable, and sustainable disaster response while fostering strong inter-organizational coordination, which aligns with IESS objectives.

3.2. Emergency supply capacity

Emergency supplies capacity primarily focuses on the essential characteristics of supplies that emergency suppliers can provide (Zhang et al., 2022). This capacity can be broadly categorized into availability (C6), quality (C7), and cost-effectiveness (C8) (Li et al., 2022a; Pamucar et al., 2022).

- **Supply availability (C6)**: Supply availability ensures the prompt provision and acquisition of essential supplies during disasters or emergencies, enabling rapid support to affected areas (Ge et al., 2020).
- **Supply quality (C7)**: Supply quality refers to the durability and performance of supplies under challenging conditions, with high-quality supplies ensuring reliability and stability, while low-quality ones may deteriorate, hindering rescue efforts and increasing risks (Afrasiabi et al., 2022).
- **Supply cost-effectiveness (C8)**: Supply cost-effectiveness involves procurement and usage costs, with low cost-effectiveness potentially limiting supply acquisition and rescue operations, while identifying cost-effective supplies and strategies helps overcome financial constraints during emergencies (Liu et al., 2022).

Table 2
Notations of OPA-P.

Type	Notation	Definition
Set	$\mathcal{I}$	Set of alternatives $i \in \mathcal{I} := 1, \dots, I$.
	$\mathcal{J}$	Set of attributes $j \in \mathcal{J} := 1, \dots, J$.
	$\mathcal{K}$	Set of experts $k \in \mathcal{K} := 1, \dots, K$.
Parameter	re_k	Rank of expert $k \in \mathcal{K}$ and $re_k \in [K]$.
	rc_{jk}	Rank of attribute $j \in \mathcal{J}$ given by expert $k \in \mathcal{K}$ and $rc_{jk} \in [J]$.
	ra_{ijk}	Rank of alternative $i \in \mathcal{I}$ under attribute $j \in \mathcal{J}$ given by expert $k \in \mathcal{K}$ and $ra_{ijk} \in [I]$.
Variable	Z	Objective function of OPA-P.
	w_{ijk}	Decision weight of alternative $i \in \mathcal{I}$ under attribute $j \in \mathcal{J}$ given by expert $k \in \mathcal{K}$.
Partial-order theory symbol	$(\mathcal{I}, \preceq_{POCT})$	Partial-order cumulative transformation set (POCTS).
	$\mathcal{I}_{i_1}^{+POCT}$	Upper set of alternative $i_1 \in \mathcal{I}$ of POCTS.
	$\mathcal{I}_{i_1}^{-POCT}$	Lower set of alternative $i_1 \in \mathcal{I}$ of POCTS.
	$\mathcal{I}_{i_1}^{\neq POCT}$	Incomparable set of alternative $i_1 \in \mathcal{I}$ of POCTS.
	$\mathbf{P}^{POCT}$	POCTS in binary matrix form.
	$\mathbf{S}^{POCT}$	General skeleton matrix of POCTS.

These supply-related attributes ensure that the IESS can effectively mobilize resources, balancing quality, cost, and availability to maximize impact in disaster scenarios, which aligns with IESS objectives.

4. Partial ordinal priority approach

OPA-P extends OPA with a partial-order structure based on partial-order theory and graph theory. It consists of two main components: the first is weight elicitation through a linear programming problem based on ordinal preference information; the second is identification of Pareto-optimal alternatives through partial-order cumulative transformation and the generation of an adversarial Hasse diagram. Table 2 outlines the notations used in OPA-P.

4.1. Preliminaries

Definition 1 (Partial-order relation). Let $\mathcal{R}$ be a binary relation on a set $\mathcal{X}$, denoted as $\mathcal{R} \subseteq \mathcal{X} \times \mathcal{X}$ (i.e., $\mathcal{R}$ is a subset of the Cartesian product of $\mathcal{X}$). The relation $\mathcal{R}$ is defined as a partial-order relation on the set $\mathcal{X}$, denoted as $\preceq$, if it satisfies:

- **Reflexive relation:** $x \preceq x$ for all $x \in \mathcal{X}$.
- **Anti-symmetric relation:** $(x \preceq y) \wedge (y \preceq x)$ implies $x = y$ for all $x, y \in \mathcal{X}$.
- **Transitive relation:** $(x \preceq y) \wedge (y \preceq z)$ implies $x \preceq z$ for all $x, y, z \in \mathcal{X}$.

Definition 2 (Total-order relation). A partial-order relation $\mathcal{R}$ on a set $\mathcal{X}$ is defined as total-order relation if it satisfies the strong completeness (i.e., $(x \preceq y) \vee (y \preceq x)$ for all $x, y \in \mathcal{X}$).

The partial-order relation is more "flexible" than the total-order relation, allowing for cases where alternatives are either equivalent or incomparable. This study examines the partial-order and total-order relations on the alternative set $\mathcal{I}$ based on their performance with respect to the attribute set $\mathcal{J}$ under expert evaluations from the set $\mathcal{K}$, denoted as the partial-order set $(\mathcal{I}, \preceq_{\mathcal{J}|\mathcal{K}})$ and total-order set $(\mathcal{I}, \leq_{\mathcal{J}|\mathcal{K}})$, respectively.

Definition 3 (Lower and upper set of partial-order set). For all $x \in \mathcal{X}$, the lower and upper sets of x on the partial-order set $(\mathcal{X}, \preceq)$ are defined as $\mathcal{X}_x^- := \{y \in \mathcal{X} : y \preceq x\}$ and $\mathcal{X}_x^+ := \{y \in \mathcal{X} : x \preceq y\}$.

Lemma 1. *Given a partial-order set $(\mathcal{X}, \preceq)$, the following statement holds:*

(1) $x \in \mathcal{X}_y^- \Leftrightarrow y \in \mathcal{X}_x^+$;
(2) $x \preceq y \Leftrightarrow \mathcal{X}_x^- \subseteq \mathcal{X}_y^-$ *for all* $x, y \in \mathcal{X}$.

Definition 4 (Order-preserving mapping of partial-order set). A mapping $f : \mathcal{X} \mapsto \mathcal{Y}$ is defined as order-preserving mapping from $(\mathcal{X}, \preceq_1)$ to $(\mathcal{Y}, \preceq_2)$ if $x \preceq_1 y \Rightarrow f(x) \preceq_2 f(y)$ for all $x, y \in \mathcal{X}$.

Definition 5 (Inclusion relation in partial-order set). Let $\mathcal{X}_{x,1}^-$ and $\mathcal{Y}_{x,2}^-$ be the lower set of x on the partial-order set $(\mathcal{X}, \preceq_1)$ and $(\mathcal{Y}, \preceq_2)$, respectively. If $\mathcal{X}_{x,1}^- \subseteq \mathcal{Y}_{x,2}^-$ holds for all $x \in \mathcal{X}$, then $(\mathcal{X}, \preceq_1)$ is a subset of $(\mathcal{Y}, \preceq_2)$, denoted as $(\mathcal{X}, \preceq_1) \subseteq (\mathcal{Y}, \preceq_2)$.

Theorem 1. *Given $(\mathcal{X}, \preceq_1) \subseteq (\mathcal{Y}, \preceq_2)$ and $\mathcal{X} = \mathcal{Y}$, if $x \preceq_1 y$ for all $x, y \in \mathcal{X}$, then $x \preceq_2 y$.*

Theorem 1 implies that when two partial-order sets, sharing the same alternatives yet differing in attributes, exhibit an inclusive relation, the evaluation outcomes are order-preserving in the context of MADM. In addition, combining Lemma 1, it can be inferred that if two alternatives are comparable, no matter how the weights of experts and attributes change, the partial-order relation between the two alternatives remains unchanged. The above insight will serve as the basis for constructing the partial-order set based on partial-order cumulative transformation in OPA-P that incorporates the information of attribute weight.

4.2. Weight optimization based on ordinal preference information

This section derives a decision weight optimization model based on ordinal preference information, eliminating the need of data standardization and expert opinion aggregation. By convention, the most important object is ranked 1, followed by others in descending order. In OPA-P, the decision-maker first assigns the ranking $re_k \in [K]$ to each expert $k \in \mathcal{K}$. Subsequently, each expert $k \in \mathcal{K}$ independently provides the ranking $rc_{jk} \in [J]$ to attribute $j \in \mathcal{J}$ and the ranking $ra_{ijk} \in [I]$ to alternative $i \in \mathcal{I}$ under attribtue $j \in \mathcal{J}$. For clarity, define the following three sets:

$$\mathcal{R}^1 := \{(h,i,j,k) \in \mathcal{I} \times \mathcal{I} \times \mathcal{J} \times \mathcal{K} : ra_{hjk} = r_{ijk} + 1, r_{ijk} \in [I-1]\},$$
$$\mathcal{R} := \{(i,j,k) \in \mathcal{I} \times \mathcal{J} \times \mathcal{K}\},$$
$$\mathcal{R}^2 := \{(i,j,k) \in \mathcal{I} \times \mathcal{J} \times \mathcal{K} : r_{ijk} = I\},$$

where $\mathcal{R}$ represents the index set of all experts, attributes, and alternatives, $\mathcal{R}^1$ represents the set of alternatives with consecutive rankings across all experts and attributes, and $\mathcal{R}^2$ denotes the set of alternatives ranked last by all experts and attributes. Let $A_{ijk}^{ra_{ijk}}$ denote alternative i ranked ra_{ijk} under attribute j by expert k, with weight w_{ijk}. Under attribute j, if expert k prefers alternative i over alternative h, then the weight of alternative i is greater than that of alternative h, i.e.,

$$A_{hjk}^{ra_{hjk}} \prec_{\mathcal{J}|\mathcal{K}} A_{ijk}^{ra_{ijk}} \Leftrightarrow w_{hjk} < w_{ijk} \Leftrightarrow 0 < w_{ijk} - w_{hjk}, \qquad \forall (h,i,j,k) \in \mathcal{R}^1,$$
$$0 \prec_{\mathcal{J}|\mathcal{K}} A_{ijk}^{ra_{ijk}} \Leftrightarrow 0 < w_{ijk}, \qquad \forall (i,j,k) \in \mathcal{R}^2,$$

or equivalently, for any $(j,k) \in \mathcal{J} \times \mathcal{K}$,

$$I \text{ terms} \begin{cases} w_{ijk}^{ra_{ijk}=1} - w_{hjk}^{ra_{hjk}=2} > 0, \\ w_{ijk}^{ra_{ijk}=2} - w_{hjk}^{ra_{hjk}=3} > 0, \\ \vdots \\ w_{ijk}^{ra_{ijk}=r} - w_{hjk}^{ra_{hjk}=r+1} > 0, \\ \vdots \\ w_{ijk}^{ra_{ijk}=I-1} - w_{hjk}^{ra_{hjk}=I} > 0, \\ w_{ijk}^{ra_{ijk}=I} > 0. \end{cases}$$

To evaluate the impact of ordinal preference information on the weight of the alternatives, OPA-P incorporates re_k, rc_{jk}, and ra_{ijk} in evaluating weight disparities between alternatives with consecutive rankings. Both sides of the inequality are multiplied by the ranking parameters:

$$\begin{aligned} & re_k rc_{jk} ra_{ijk}(w_{ijk} - w_{hjk}) > 0, \qquad \forall (h,i,j,k) \in \mathcal{R}^1, \\ & re_k rc_{jk} ra_{ijk}(w_{ijk}) > 0, \qquad \forall (i,j,k) \in \mathcal{R}^2. \end{aligned} \tag{1}$$

Intuitively, this manipulation to the intuition that as rankings increase, the marginal effect of weight disparity between consecutive alternatives diminishes, which is proved by Wang (2024a) via deriving an equivalent formulation and analyzing the decomposability of the results. The above discussion present a logic for analyzing the weight disparities of alternatives with consecutive rankings assigned by experts, which can also be extended to the analysis of experts and attributes. Notably, decision-makers seek weight computations that align with the preferences of experts and maximize discrimination, a common robust approach in MADM literature (Grabisch, Kojadinovic, & Meyer, 2008; Li et al., 2023; Lu, Wu, Deng, & Li, 2023). Therefore, OPA-P formulates a multi-objective optimization model for decision weight optimization within a normalized weight space based on ordinal preference information, as shown in Eq. (2).

$$\begin{aligned} \max_{\boldsymbol{w}} & \begin{cases} re_k rc_{jk} ra_{ijk}(w_{ijk} - w_{hjk}) & \forall (h,i,j,k) \in \mathcal{R}^1 \\ re_k rc_{jk} ra_{ijk}(w_{ijk}) & \forall (i,j,k) \in \mathcal{R}^2 \end{cases} \\ \text{s.t.} & \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} w_{ijk} = 1 \\ & w_{ijk} \geq 0, \qquad \forall (i,j,k) \in \mathcal{R} \end{aligned} \tag{2}$$

Then, the maxmin method is utilized to solve the above multi-objective optimization problem:

$$\begin{aligned} \max_{\boldsymbol{w}} \min_{(i,j,k)\in\mathcal{R}} & \begin{cases} re_k rc_{jk} ra_{ijk}(w_{ijk} - w_{hjk}), & \forall (h,i,j,k) \in \mathcal{R}^1, \\ re_k rc_{jk} ra_{ijk}(w_{ijk}), & \forall (i,j,k) \in \mathcal{R}^2, \end{cases} \\ \text{s.t.} & \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} w_{ijk} = 1, \\ & w_{ijk} \geq 0, \qquad \forall (i,j,k) \in \mathcal{R}. \end{aligned} \tag{3}$$

Let

$$z = \min_{(i,j,k)\in\mathcal{R}} \begin{cases} re_k rc_{jk} ra_{ijk}(w_{ijk} - w_{hjk}), & \forall (h,i,j,k) \in \mathcal{R}^1, \\ re_k rc_{jk} ra_{ijk}(w_{ijk}), & \forall (i,j,k) \in \mathcal{R}^2. \end{cases} \tag{4}$$

Substituting Eq. (4) into Eq. (3), the above maxmin problem can be can be further transformed into a linear programming problem, which gives the decision weight optimization model of OPA-P based on ordinal preference in Eq. (5).

$$\begin{aligned} \max_{\boldsymbol{w},z} \ & z \\ \text{s.t.} \ & z \leq re_k rc_{jk} ra_{ijk}(w_{ijk} - w_{hjk}) \qquad \forall (h,i,j,k) \in \mathcal{R}^1 \\ & z \leq re_k rc_{jk} ra_{ijk}(w_{ijk}) \qquad \forall (i,j,k) \in \mathcal{R}^2 \\ & \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} w_{ijk} = 1 \\ & w_{ijk} \geq 0 \qquad \forall (i,j,k) \in \mathcal{R} \end{aligned} \tag{5}$$

After obtaining the optimal solution z^* and w_{ijk}^* for all $(i,j,k) \in \mathcal{R}$, the weights of experts, attributes, and alternatives can be calculated by:

$$\begin{aligned} w_i^{\mathcal{I}} &= \sum_{j=1}^{J} \sum_{k=1}^{K} w_{ijk}^*, \qquad \forall i \in \mathcal{I}, \\ w_j^{\mathcal{J}} &= \sum_{i=1}^{I} \sum_{k=1}^{K} w_{ijk}^*, \qquad \forall j \in \mathcal{J}, \\ w_k^{\mathcal{K}} &= \sum_{i=1}^{I} \sum_{j=1}^{J} w_{ijk}^*, \qquad \forall k \in \mathcal{K}. \end{aligned} \tag{6}$$

However, the above computed alternative weights project numerous attributes onto a single comprehensive attributes (He et al., 2021; Lu et al., 2023; Yue, Lu, & Shi, 2022). Consequently, the outcomes lack stability in addressing situations involving Pareto-optimal solutions within the alternatives, which is a common challenge with the most MADM methods (Cao, Hu, & Yue, 2023; He et al., 2021; Li et al., 2023). To address this limitation, in this study, the outcomes of weight optimization based on ordinal preference information will undergo a partial-order cumulative transformation in OPA-P, which incorporates information of attribute weight.

4.3. Partial-order cumulative transformation

The partial-order cumulative transformation of alternative weights in OPA-P primarily aims to construct a partial-order set incorporating the information of attribute weight. The core of partial-order cumulative transformation lies in ensuring the newly constructed partial-order set has the property of order-preserving.

The first step is to compute alternative weights under the attributes:

$$w_{ij}^{\mathcal{I}\mathcal{J}} = \sum_{k=1}^{K} w_{ijk}^*, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{J}. \tag{7}$$

Denote $(\mathcal{I}, \leq_{AC})$ as the partial-order set derived from the strict Pareto-optimality condition based on $\mathbf{W}^{\mathcal{I}\mathcal{J}}$ and $(\mathcal{I}, \leq_{SPCA})$ as the total-order set derived from $\boldsymbol{w}^{\mathcal{I}}$ (i.e., single projected comprehensive attribtue). The partial-order relation of the strict Pareto-optimality condition emphasizes the strict dominance relation between alternatives on each attribute, while the total-order relation $(A, \leq_{SPCC})$ is of a stronger property in which each pair of alternatives is comparable. In this study, the derived partial-order set is theoretically shown to be more flexible than the partial-order set derived from the strict Pareto-optimality condition over each attribute, and more robust than the total-order set based on the single projected comprehensive attribute computed by decision weight optimization model.

Definition 6 (Partial-order cumulative transformation set)**.** A partial-order set is defined as the partial-order cumulative transformation set (POCTS) corresponding to $(\mathcal{I}, \preceq_{AC})$, denoted as $(\mathcal{I}, \preceq_{POCT})$, if it is derived from the partial-order cumulative transformation weight given by:

$$w_{i(j)}^{POCT} = \sum_{j=1}^{o} w_{i(j)}^{\mathcal{I}\mathcal{J}}, \quad \forall (i,(j)) \in \mathcal{I} \times \mathcal{J}, o = 1, \ldots, J, \tag{8}$$

where the attributes in $\mathbf{W}^{\mathcal{I}\mathcal{J}}$ are arranged in descending order indexed by $(j) \in \mathcal{J}$.

The partial-order cumulative transformation weight can be expressed in matrix form:

$$\mathbf{W}^{POCT} = \mathbf{W}^{\mathcal{I}\mathcal{J}}\mathbf{H} = \begin{bmatrix} w_{1(1)}^{\mathcal{I}\mathcal{J}} & w_{1(1)}^{\mathcal{I}\mathcal{J}} + w_{1(2)}^{\mathcal{I}\mathcal{J}} & \cdots & w_{1(1)}^{\mathcal{I}\mathcal{J}} + w_{1(2)}^{\mathcal{I}\mathcal{J}} + \cdots + w_{1(J)}^{\mathcal{I}\mathcal{J}} \\ w_{2(1)}^{\mathcal{I}\mathcal{J}} & w_{2(1)}^{\mathcal{I}\mathcal{J}} + w_{2(2)}^{\mathcal{I}\mathcal{J}} & \cdots & w_{2(1)}^{\mathcal{I}\mathcal{J}} + w_{2(2)}^{\mathcal{I}\mathcal{J}} + \cdots + w_{2(J)}^{\mathcal{I}\mathcal{J}} \\ \vdots & \vdots & \ddots & \vdots \\ w_{I(1)}^{\mathcal{I}\mathcal{J}} & w_{I(1)}^{\mathcal{I}\mathcal{J}} + w_{I(2)}^{\mathcal{I}\mathcal{J}} & \cdots & w_{I(1)}^{\mathcal{I}\mathcal{J}} + w_{I(2)}^{\mathcal{I}\mathcal{J}} + \cdots + w_{I(J)}^{\mathcal{I}\mathcal{J}} \end{bmatrix}, \tag{9}$$

where $\mathbf{H}$ is the upper triangular matrix, which implies a linear mapping, incorporating weight information, of cumulative transformations applied to $\mathbf{W}^{\mathcal{I}\mathcal{J}}$.

Comparing the row vectors of $\mathbf{W}^{POCT}$ yields the POCTS in binary matrix form among the alternatives $\mathbf{P}^{POCT} \in \mathbb{R}^{I \times I}$ with its elements given by, for all $i_1, i_2 \in \mathcal{I}$,

$$p_{i_1 i_2}^{POCT} = \begin{cases} 1, & \text{if } w_{i_1 j}^{POCT} \geq w_{i_2 j}^{POCT}, \forall j \in \mathcal{J}, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

By Definition 3, for any $i_1 \in \mathcal{I}$, the lower and upper set of POCTS in binary matrix form can be determined by:

$$\begin{aligned} \mathcal{I}_{i_1}^{-POCT} &:= \left\{ i_2 \in \mathcal{I} : p_{i_1 i_2}^{POCT} = 1 \right\}, \\ \mathcal{I}_{i_1}^{+POCT} &:= \left\{ i_2 \in \mathcal{I} : p_{i_2 i_1}^{POCT} = 1 \right\}. \end{aligned} \tag{11}$$

The following Theorems 2 and 3 give the order-preserving property of the proposed POCTS incorporating information of attribute weights.

Theorem 2. *Let $(\mathcal{I}, \preceq_{POCT})$ be the POCTS of $(\mathcal{I}, \preceq_{AC})$ derived from $\mathbf{W}^{\mathcal{I}\mathcal{J}}$. Then $(\mathcal{I}, \preceq_{AC}) \subseteq (\mathcal{I}, \preceq_{POCT})$.*

Theorem 3. *Let $(\mathcal{I}, \preceq_{POCT})$ be the POCTS of $(\mathcal{I}, \preceq_{AC})$ derived from $\mathbf{W}^{\mathcal{I}\mathcal{J}}$. For all $i_1, i_2 \in \mathcal{I}$, if $i_1 \preceq_{AC} i_2$, then $w_{i_1 j}^{POCT} \leq w_{i_2 j}^{POCT}$.*

Notably, the last column in $\mathbf{W}^{POCT}$ equals to $\boldsymbol{w}^{\mathcal{I}}$ computed by decision weight optimization model in Eq. (6). According to Definition 2, $(\mathcal{I}, \preceq_{POCT}) \subseteq (\mathcal{I}, \preceq_{SPCC})$. Therefore, $(\mathcal{I}, \preceq_{AC}) \subseteq (\mathcal{I}, \preceq_{POCT}) \subseteq (\mathcal{I}, \preceq_{SPCC})$, which gives the relationship among the proposed POCTS, the partial-order set based on the strict Pareto-optimality condition across all attributes, and the total-order set based on the single projected comprehensive attribute. This indicates that the partial-order set derived from partial-order cumulative transformation balances the total-order set based on comprehensive evaluation weights and the partial-order set based on strict Pareto-optimality, thus creating a more robust partial-order relation. From a MADM perspective, the POCTS suggests that alternatives facing a disadvantage in a significant attribute may still be considered viable, provided that subsequent attributes can offset the shortcoming in that specific attribute. This ensures a more stable and reliable outcome for decision-makers when selecting the optimal alternative.

However, it is evident that redundant information exists in the generated POCTS binary matrix, which can be streamlined by leveraging the transmissibility property of the partial-order relation. This refined dominance structure will offer visual support to decision-maker in selecting the optimal alternative.

4.4. Adversarial hasse diagram generation

In order to streamline the reduction information in partial-order relation, this section proposes the adversarial Hasse diagram of OPA-P, drawing inspiration from the Hasse diagram (Yue et al., 2022) and the adversarial interpretative structural modeling (Su, Woo, Chen, & Park, 2023; Wang, Cui, & Gao, 2024). Compared to the conventional Hasse diagram, the adversarial Hasse diagram not only identifies the most simplified dominance relations but also extracts hierarchical dominance structures of alternatives based on the non-dominant ascending (NDA) and descending (NDD) rules. These dual-direction extraction rules empower decision-maker with a more comprehensive perspective for decision-making and the dominance structure of alternatives in adversarial Hasse diagram offers intuitive insights into Pareto-optimal alternatives, and clustering hierarchy information of alternatives.

Let $\mathcal{I}_l$ represent the set of alternatives at layer $l \in \mathcal{L} := \{1, \ldots, L\}$ in the adversarial Hasse diagram. Let $\mathbf{P}_{-\mathcal{I}'}^{POCT}$ represent the matrix obtained by removing row and column $i' \in \mathcal{I}'$ from $\mathbf{P}^{POCT}$. The following Algorithm 1 outlines the procedure to generate the adversarial Hasse diagram of OPA-P based on the lower and upper set of POCTS.

Algorithm 1 Adversarial Hasse diagram generation.

1: **Input:** alternative set $\mathcal{I}$, POCTS in binary matrix form $\mathbf{P}^{POCT}$, and unit matrix $\mathbf{I}$.
2: **Output:** Alternative set $\mathcal{I}_l^A$ or $\mathcal{I}_l^D$ for each layer $l \in \mathcal{L}$ and general skeleton matrix $\mathbf{S}^{POCT}$.
3: **Initialization:** $\bar{\mathcal{I}} = \mathcal{I}$ and $l = 1$.
4: Set $\mathbf{S}^{POCT} = (\mathbf{P}^{POCT} + \mathbf{I})^\top - ((\mathbf{P}^{POCT})^\top)^2 - \mathbf{I}$.
5: **while** $\bar{\mathcal{I}} \neq \emptyset$ **do**
6: Set $\mathcal{I}_l := \emptyset$.
7: **for** $\forall i_1 \in \bar{\mathcal{I}}$ **do**
8: Determine $\mathcal{I}_{i_1}^{-POCT}$ and $\mathcal{I}_{i_1}^{+POCT}$ according to Eq. (11).
9: **if** using non-dominant ascending rule **then**
10: Set $\mathcal{I}_l = \mathcal{I}_l \cup \arg\{(\mathcal{I}_{i_1}^{-POCT} \cup \{i_1\}) \cap (\mathcal{I}_{i_1}^{+POCT} \cup \{i_1\}) = (\mathcal{I}_{i_1}^{-POCT} \cup \{i_1\})\}$.
11: **else if** using non-dominant descending rule **then**
12: Set $\mathcal{I}_l = \mathcal{I}_l \cup \arg\{(\mathcal{I}_{i_1}^{-POCT} \cup \{i_1\}) \cap (\mathcal{I}_{i_1}^{+POCT} \cup \{i_1\}) = (\mathcal{I}_{i_1}^{+POCT} \cup \{i_1\})\}$.
13: **end if**
14: **end for**
15: Set $\bar{\mathcal{I}} = \bar{\mathcal{I}} \backslash \mathcal{I}_l$ and $\mathbf{P}^{POCT} = \mathbf{P}_{-\mathcal{I}_l}^{POCT}$.
16: $l = l + 1$.
17: **end while**
18: Set $L = l$.
19: **return** $\mathbf{S}^{POCT}$ and $\mathcal{I}_l$ for $l = 1, \ldots, L$.

Line 4 calculates the general skeleton matrix with the time complexity of $\mathcal{O}(I^3)$, while Lines 5–17 perform the hierarchical classification of alternatives with the time complexity of $\mathcal{O}(I^2 L)$. Thus, the time complexity of Algorithm 1 is $\mathcal{O}(I^3) = \max\{\mathcal{O}(I^2 L), \mathcal{O}(I^3)\}$, which is from the fact that $L \leq I$, where the equality gives a total-order relation. The alternative sets derived from NDA are arranged from top to bottom based on index l, while those derived from the NDD are arranged from bottom to top. Connecting the alternatives within the extracted dominant hierarchies, based on NDA and NDD rules, and the general skeleton matrix, generates the adversarial Hasse diagram of OPA-P. In this diagram, top-layer alternatives identified through NDA and NDD represent Pareto-optimal solutions. The number of dominant hierarchies, determined by these rules, is the same, with at least one consistent alternative per layer. The dominance relation within the general skeleton matrix is transitive, and alternatives within the same layer are incomparable. These properties of the adversarial Hasse diagram assist decision-makers in analyzing OPA-P results.

4.5. Implementation steps

This section outlines the implementation steps, notes, and algorithmic complexity analysis of OPA-P. The following Procedure 1 presents the implementation steps of OPA-P. When implementing OPA-P, it is important to note that OPA-P allows for the occurrence of tied rankings, where the weight difference between tied alternatives is zero. In individual decision-making, the decision-weight elicitation model in Eq. (5) can be formulated without the parameters and constraints related to multiple experts.

Procedure 1 Implementation steps of OPA-P.

1: **Step 1: Identify the elements of decision-making.**
2: Determine the expert set $\mathcal{K}$ and attribute set $\mathcal{I}$ involved in the decision-making process.
3: Identify the attribute set $\mathcal{J}$ according to the decision objectives.
4: **Step 2: Obtain the input data.**
5: Assign important ranking re_k for each expert $k \in \mathcal{K}$.
6: **for** all expert $k \in \mathcal{K}$ **do**
7: Assign important ranking rc_{jk} for each attribute $j \in \mathcal{J}$.
8: Assign important ranking ra_{ijk} for each alternative $i \in \mathcal{I}$ under each attribute $j \in \mathcal{J}$.
9: **end for**
10: **Step 3: Determine the decision weights.**
11: Solve Eq. (5) for decision weight w^*_{ijk} for all alternative $i \in \mathcal{I}$ across attribute $j \in \mathcal{J}$ under the evaluation of expert $k \in \mathcal{K}$.
12: Calculate the expert weights $\boldsymbol{w}^{\mathcal{K}}$, attribute weights $\boldsymbol{w}^{\mathcal{J}}$, and alternative weights $\boldsymbol{w}^{\mathcal{I}}$ by Eq. (6).
13: **Step 4: Determine the partial-order cumulative transformation set.**
14: Calculate the weight $w^{\mathcal{I}\mathcal{J}}_{ij}$ for all alternative $i \in \mathcal{I}$ across attributes $j \in \mathcal{J}$ according to Eq. (7).
15: Calculate the matrix of partial-order cumulative transformation weight $\mathbf{W}^{POCT}$ according to Eq. (9).
16: Determine the partial-order cumulative transformation set in binary matrix form $\mathbf{P}^{POCT}$ according to Eq. (10).
17: **Step 5: Generate adversarial Hasse diagram.**
18: Generate adversarial Hasse diagram according to Algorithm 1.

Although OPA-P offers benefits in Pareto-optimal analysis, data accessibility, and the elimination of data standardization and expert opinion aggregation, it is essential to evaluate the complexity of the proposed method to ensure its computational feasibility in decision-making. Specifically, the decision-weight elicitation model in Eq. (5) is a standard linear programming problem with $IJK+1$ variables, resulting in a worst-case time complexity of $\mathcal{O}(2^{IJK+1})$, which is exponential. However, Wang (2024a) demonstrated that this problem has a closed-form solution, reducing the time complexity to $\mathcal{O}(IJK)$. The time complexities for calculating the weights of experts, attributes, and alternatives are $\mathcal{O}(I)$, $\mathcal{O}(J)$, and $\mathcal{O}(K)$, respectively. Therefore, the time complexity for Step 3 of OPA-P is $\mathcal{O}(IJK)$. The time complexities for Lines 16 and 17 are $\mathcal{O}(IJ)$ and $\mathcal{O}(I^2J)$, respectively, making the time complexity for Step 4 of OPA-P $\mathcal{O}(I^2J)$. Considering the worst-case time complexity of Algorithm 1 is $\mathcal{O}(I^3)$, the time complexity of OPA-P is $\max\{\mathcal{O}(IJK), \mathcal{O}(I^2J), \mathcal{O}(I^3)\}$, which can be solved in polynomial time. Therefore, since MADM generally deals with small-scale discrete sets of alternatives, OPA-P is computationally practical. Table 3 compares the time complexity of OPA-P with that of other traditional MADM methods, considering the same number of experts, attributes, and alternatives, and assuming that the expert and attribute weights are predefined and the number of alternatives is larger than the number of attributes and experts. The corresponding codes of these MADM methods are available at https://github.com/Valdecy/pyDecision. However, it is important to note that OPA-P is an integrated method that does not require predefined expert and attribute weights.

5. Case study

5.1. Case description and data collection

This section uses the IESS of the 7.20 mega-rainstorm disaster in Zhengzhou, China, as a case to demonstrate the proposed OPA-P. The catastrophic scale of the rainfall, which broke historical records and overwhelmed existing flood control and drainage systems, caused severe waterlogging and widespread flooding. This led to an urgent need for the mobilization of medical supplies, food, and other essential relief materials (Peng & Zhang, 2022). The disaster's unprecedented intensity exceeded the capacity of pre-identified suppliers, requiring rapid adjustments to the provisioning network. In such a crisis, the deployment of IESS is essential for efficiently coordinating relief efforts amidst tight deadlines, information uncertainty, and the involvement of multiple stakeholders. This case study demonstrates how the OPA-P framework can effectively manage complex, multi-stakeholder scenarios during large-scale emergencies, where traditional methods often fall short. By leveraging the flexibility and adaptability of the OPA-P, this study underscores its practical applicability in real-world crisis management, offering a significant enhancement over conventional resource management approaches.

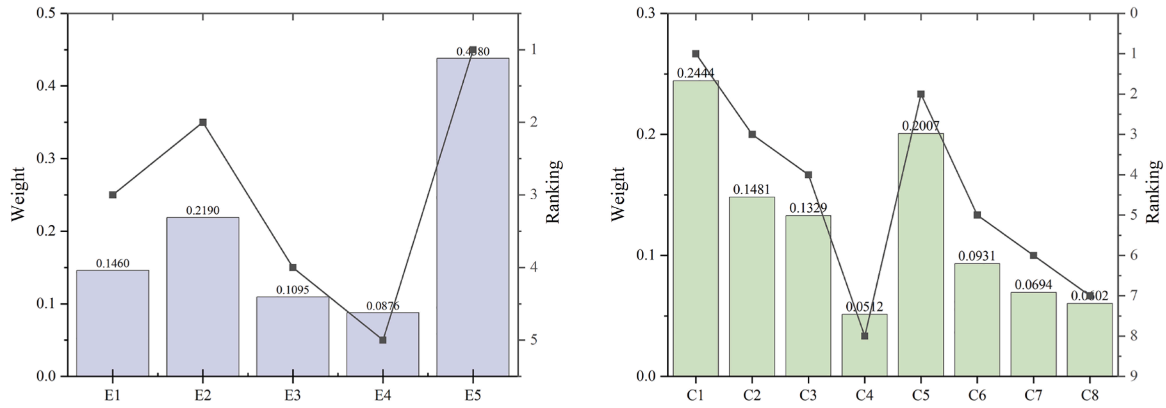
Fifteen emergency suppliers, designated as A1 to A15, are at the disposal for selection within the stricken region. These suppliers encompass a spectrum of characteristics, including location, responsiveness, and supply capacity, each exhibiting distinct variations. Certain entities prioritize a stable supply chain and swift responsiveness, albeit at a cost premium beyond the norm. Conversely, other entities boast advantageous geographical placement and efficient traffic connectivity, expediting the provision of flood relief in times of crisis. However, these advantages might be counterbalanced by uncertainties surrounding the quality and durability of the provided rescue materials. These distinct characteristics will serve as benchmarks aiding experts in ranking alternatives. Following this, five experts from various departments act as representatives for stakeholders in the selection of emergency suppliers. This group includes representatives from the Zhengzhou Emergency Management Department, the Zhengzhou Civil Affairs Department, and the Zhengzhou Municipal Health Commission. The experts have been prioritized based on their authority in emergency response decision-making, with E5 having the highest rank, followed by E2, E1, E3, and E4 in descending order (E5 > E2 > E1 > E3 > E4). These experts evaluate the essential requirements for specifying emergency suppliers and provide rankings for both attributes and suppliers, as detailed in Table B.1.

5.2. Result analysis

Fig. 2 displays the weights of experts and attributes with the results presented in Table B.2. Regarding expert weights, E5, possessing the ut-

Table 3
Time complexity comparison.

	OPA-P	TOPSIS	VIKOR	COPRAS	CRADIS	MABAC	EDAS	MOORA
Complexity	$\mathcal{O}(I^3)$	$\mathcal{O}(IJK)$	$\mathcal{O}(I^2K)$	$\mathcal{O}(IJK)$	$\mathcal{O}(IJK)$	$\mathcal{O}(I^2JK)$	$\mathcal{O}(IJK)$	$\mathcal{O}(IJK)$



(a) Results of experts (b) Results of attributes

Fig. 2. Optimal weights and rankings of experts and attributes of the case IESS.

most authority, is assigned 0.4380, followed by E2 with 0.2190. Subsequently, E1, E3, and E4 trail behind with the weights of 0.1460, 0.1095, and 0.0876, respectively. Notably, the results of expert weights reveal a distinct trend of diminishing marginal effects. Regarding attributes, the most critical factor is the response speed (C1) with a weight of 0.2444. Closely followings are collaborative experience and credibility(C5), delivery reliability (C2), and geographic coverage (C3), with weights of 0.2007, 0.1481, and 0.1329, respectively. Conversely, the weighs for supply availability (C6), supply quality (C7), supply cost-effectiveness (C8), and supplier sustainability (C4) are relatively lower, with weights of 0.0931, 0.0694, 0.0602, and 0.0512, respectively.

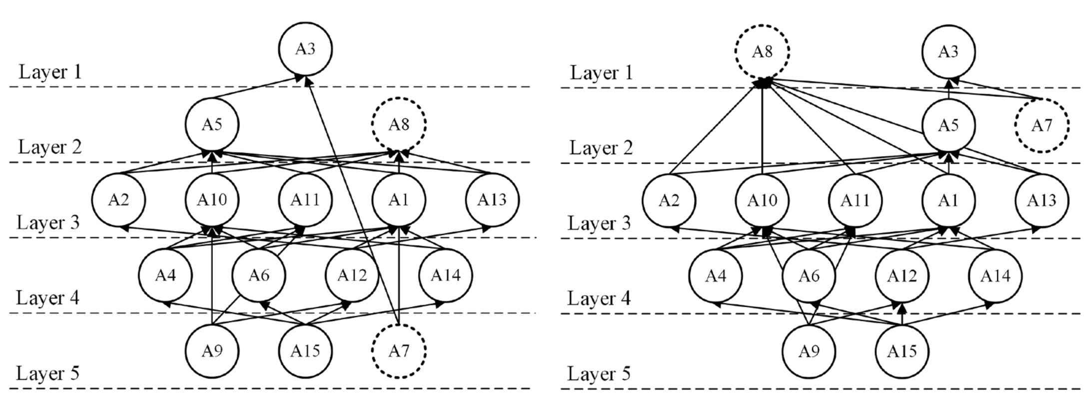
The partial-order cumulative transformation is then performed, with the transformed weights presented in Table B.3. The corresponding adversarial Hasse diagram is generated, as shown in Fig. 3,with Table 4 summarizing the hierarchical clusters of alternatives. The diagram clearly displays the IESS information regarding the Pareto-optimal and suboptimal alternatives, the dominance structure, and the hierarchical clustering details. Dashed blocks within the diagram represent the alternatives with altered hierarchies. Table 4 shows that the dominant hierarchy of the case IESS unfolds over 5 layers, elucidating the alternatives with unchanged hierarchy and altered hierarchy inherent within each layer. The structure originating from NDD positions A3 at Layer 1, succeeded by A5 and A8 at Layer 2, and A2, A10, A11, A1, and A13 at Layer 3. Layer 4 encompasses A4, A6, A12, and A14, while Layer 5, the bottom layer, contains A9, A15, and A7. Within the diagram based on NDD, the Pareto-optimal alternatives for the case IESS emerges as A3. A proximate alternative of note lies in Layer 2, represented by A5 and A8. Notably, by the transmissibility of the adversarial Haase diagram,

Table 4
Hierarchical clustering information of the case IESS.

	Alternatives with unchanged hierarchy	Alternatives with altered hierarchy (NDA)	Alternatives with altered hierarchy (NDD)
Layer 1	A3	-	A8
Layer 2	A5	A8	A7
Layer 3	A1, A2, A10, A11, A13	-	-
Layer 4	A4, A6, A12, A14	-	-
Layer 5	A9, A15	A7	-

A5 serves as a sub-optimal alternative for A3. Regarding the structure based on NDA, the dominance hierarchy of A8 has transitioned from Layer 2 to Layer 1, whereas A7 now occupies Layer 2 rather than Layer 5. This reconfiguration positions A8 and A3 in Layer 1, each with inherent merits and limitations. Furthermore, A5 and A7 are the sub-optimal alternatives for A3 and A8.

Regarding management implications, the attribute weight results of the case study reveal that during the improvisational selection of emergency suppliers, critical stakeholders prioritize suppliers' emergency response capabilities over the attributes of the supplies they offer. Notably, despite the growing focus on the sustainability of humanitarian operations in line with the UN Sustainable Development Goals, stakeholders consider the sustainability of emergency suppliers a relatively minor attribute. In the adversarial Hasse diagram based on NDD, alternative A3 emerges as the Pareto-optimal choice for the case IESS, ex-



(a) Non-dominant sinking alternative extraction (b) Non-dominant rising alternative extraction

Fig. 3. Adversarial Hasse diagram of the case IESS.

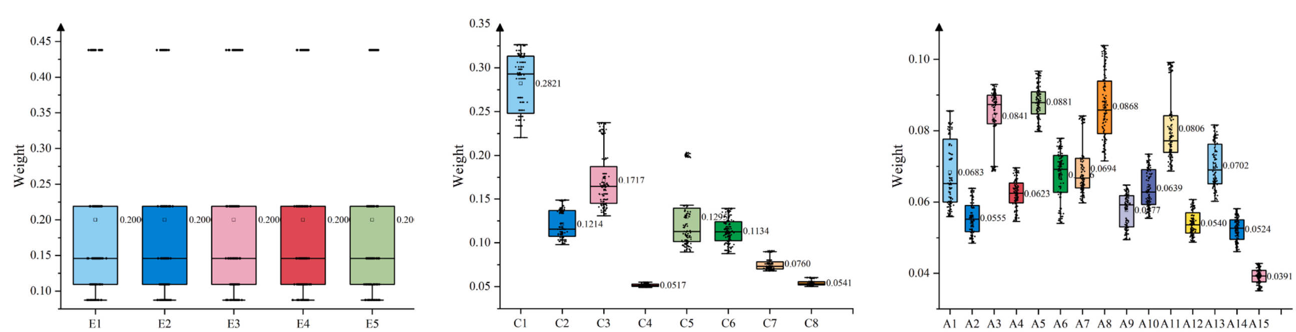
celling in supplier response speed (C1) and performing strongly across other attributes, including delivery reliability (C2) and supply quality (C7). Comparing A8 with A3 shows that A8 outperforms in supplier collaborative experience and credibility (C5) and supply cost-effectiveness (C8), while A7 surpasses A5 in C5, C2, and supplier geographic coverage (C3). These findings suggest that A7 is suitable for large-scale disasters requiring stable resource supply, and A1 is better for the scenarios with extreme time constraints. The decision-making process highlights the need to assess each emergency supplier's specific capability advantages and tailor the selection to different scenarios. For policymakers, these findings offer guidance on formulating policies that encourage suppliers to enhance their emergency response capabilities and help in selecting the most appropriate suppliers. Overall, the case study demonstrates that OPA-P equips experts with the comprehensive information on alternative dominance structures, Pareto-optimal and sub-optimal alternatives, thus enabling more transparent and robust decisions in IESS. This underscores the importance of using OPA-P for effective emergency supplier selection and improved disaster response.

5.3. Sensitivity analysis

This section performs the sensitivity analysis on input data or parameters, which is a vital numerical analysis technique to evaluate the efficacy of MADM methods. The core of OPA-P involves ordinal preference given by experts, i.e., the rankings of attributes and alternatives under attributes. However, analyzing the sensitivity of attribute and alternative rankings provided by experts involves perturbing their preference information, lacking a clear benchmark, which hinders a comprehensive analysis. Therefore, this section performs a sensitivity analysis on the expert rankings provided by decision-makers to ensure logical consistency. Specifically, a complete permutation of rankings from five experts is tested through 120 experiments. The weights assigned to experts, attributes, and alternatives by OPA-P are summarized. Additionally, the frequency with which each alternative appears as a Pareto-optimal solution is examined in the adversarial Hasse diagram. Fig. 4 and Table B.4 display box plots and descriptive statistics of the weight outcomes.

In the descriptive statistical analysis of expert weights, the mean values for five experts all demonstrate a consistent feature, standing at 0.2. The maximum and minimum values of expert weights are 0.4380 and 0.0876, respectively. This consistency is also evident across all other indicators. This outcome aligns with the intuitive observations from implementing a whole permutation experimental design. The occurrence frequency of each expert at various ranking positions is equal, resulting in the uniformity of descriptive statistical results among experts. Regarding the mean values of attribute weights, the most significant is C1, with a weight of 0.2821, followed by C3, with a weight of 0.1717. Subsequently, C5, C2, and C6 closely follow, with weights of 0.1295, 0.1214, and 0.1134, respectively, exhibiting a relatively similar trend. In contrast, the weights of C8 and C4 are the lowest, at 0.0541 and 0.0517, respectively. The Skewness results of attributes indicate that, except for C1 exhibiting left Skewness, the remaining attributes demonstrate right Skewness. Meanwhile, the Kurtosis results for the attributes reveal that all attributes exhibit negative Kurtosis, implying that the attribute weights concentrate around the mean, with relatively fewer data points in tails. Notably, the coefficients of variation for both C3 and C5 surpass the threshold (0.15), while the coefficients for other attributes remain below the threshold. Specifically, the coefficient of variation for C5 reaches 0.2956, indicating a pronounced dispersion trend. Furthermore, an examination of Fig. 4 reveals a distinct portion of C5 weights reaching 0.2033. This is primarily due to the ranking of C5 provided by expert E5, which is 1, contrasting with rankings of 4, 6, 5, and 7 from other experts. The divergent nature of these evaluations results in abnormal fluctuations in the weight of C5. Upon analyzing the mean weights of alternatives, it is evident that the top four ranked alternatives are A5, A8, A3, and A11, with weights of 0.0881, 0.0868, 0.0841, and 0.0806, respectively. Notably, the maximum weight among the alternatives is associated with A8, reaching 0.1039, while A8 exhibits a relatively high degree of dispersion. Additionally, the coefficients of variation for the weights of the alternatives do not exceed the threshold (0.15). Furthermore, both the standard deviation and variance are smaller than that of the attributes, indicating a more excellent stability of the alternatives compared to the attributes. Furthermore, it is observed that A1, A2, A5, A7, A8, A10, A11, A12, and A13 all display a negative skewness, indicating a right-skewed distribution. This implies that the weight distribution of alternatives with negative Skewness extends more gradually to the right, with the possibility of some relatively large values concentrated overall around the mean. In contrast, the remaining alternatives exhibit characteristics of a left-skewed distribution in their negative Skewness. Through statistical analysis of attribute and alternative weights, it can be deduced that the weight outcomes of OPA-P are reasonably stable.

Fig. 5 illustrates the probability of each alternative emerging as the Pareto-optimal solutions in NDA and NDD. It is evident that the probability in NDD is lower than that in NDA, consistent with the derivation of OPA-P. A3, A5, A8, A11, and A1 consistently show probabilities exceeding 10% in NDA, with A3, A5, and A8 frequently appearing as the Pareto-optimal solutions in over 40% experiments. These findings suggest that, under specific conditions, these alternatives have the potential to become the Pareto-optimal solutions. This contrasts with the weight-based total-order ranking, which focuses only on the alternative with the highest weight. The difference highlights the importance of considering the potential superiority of alternatives in various contexts, rather than relying solely on their relative positions in a weight-based total order. This underscores the reliability and validity of the adversarial Hasse diagram based on partial-order cumulative transformation of OPA-P in identifying Pareto-optimality.



(a) Results of experts (b) Results of attributes (c) Results of alternatives

Fig. 4. Box plots of the computed weight outcomes.

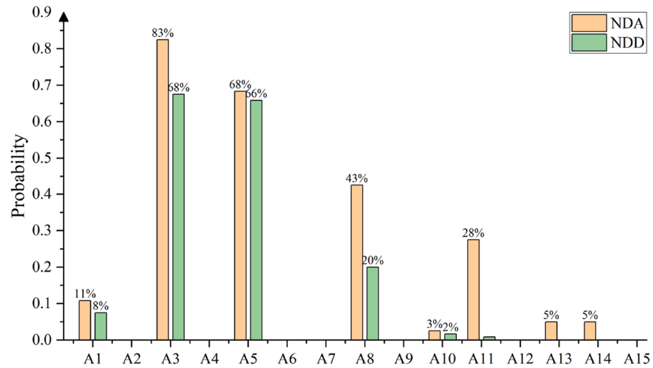


Fig. 5. Probability of attaining Pareto-optimal solutions in adversarial Hasse diagram.

5.4. Comparative analysis

This section undertakes a comparative analysis within the IESS context of the Zhengzhou mega-rainstorm disaster to validate OPA-P. Specifically, the analysis is divided into two parts: the first part compares the weight-based total-order ranking of alternatives in OPA-P with that from classical MADM methods; the second part involves perturbing expert rankings to examine the stability of the final alternative rankings obtained from different methods. This experimental design is utilized because the first part provides "reasonable" reference for comparison, with Theorems 2 and 3 indicating that the ranking results displayed in the adversarial Hasse diagram must include the weight-based total-order ranking outcomes.

In the first part, the Spearman correlation coefficient is used to evaluate the correlation between the ordinal sequences of alternative rankings:

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)},$$

where i and n are the index and number of alternatives and d_i^2 is the difference squared between alternative i in two ranking sequences. As ρ approaches 1, it indicates a strong positive correlation; as ρ approaches -1, it indicates a strong inverse correlation; and as ρ approaches 0, it suggests no significant correlation.

In the second part, perturbed expert rankings are generated from a normal distribution with a standard deviation of 1/3 and original expert rankings as the mean. This standard deviation reflects expert ranking perturbation merely without reversal, as approximately 99.7 % of data in a normal distribution lies within three standard deviations of the mean, enabling the estimation of standard deviations based on the expected perturbation interval. The following indicators are used to assess performance:

$$CR = \left\| \left(\arg\max_{r\in\mathcal{R}} p_{1r}^n, \ldots, \arg\max_{r\in\mathcal{R}} p_{Ir}^n \right)^\top - \left(r_1^*, \ldots, r_I^*\right)^\top \right\|_2,$$

$$CP = \left\| \left(\max_{r\in\mathcal{R}} p_{1r}^n, \ldots, \max_{r\in\mathcal{R}} p_{Ir}^n \right)^\top \right\|_2,$$

where n denotes the iteration step, r_i^* denotes the optimal ranking of alternative i in the first part, and p_{ir}^n denotes the probability of alternative k being ranked r. The first indicator CR measures the deviation between the final ranking and the initial reference, with smaller values being preferable, while the second indicator CP evaluates the concentration of the optimal ranking probability, with larger values being preferable.

This section selects TOPSIS, COPRAS, CRADIS, MABAC, EDAS, and MOORA as the benchmark due to their status as mainstream baseline methods in MADM of IESS. It is worth noting that ELECTRE is not selected due to the additional requirement of setting subjective threshold parameters during its implementation. Typically, the above MADM methods employ decision matrices with objective values or evaluation scores, necessitating the prior acquisition of attribute weights. Thus, $w_{ij}^{\mathcal{IJ}}$ is decomposed into the attribute weights and utilities of alternatives on attributes (i.e., $w_{ij}^{\mathcal{IJ}} = w_j^{\mathcal{J}} \times v_{ij}^{\mathcal{IJ}}$) (Wang, 2024a). Table B.5 shows the decision matrix of the selected MADM methods.

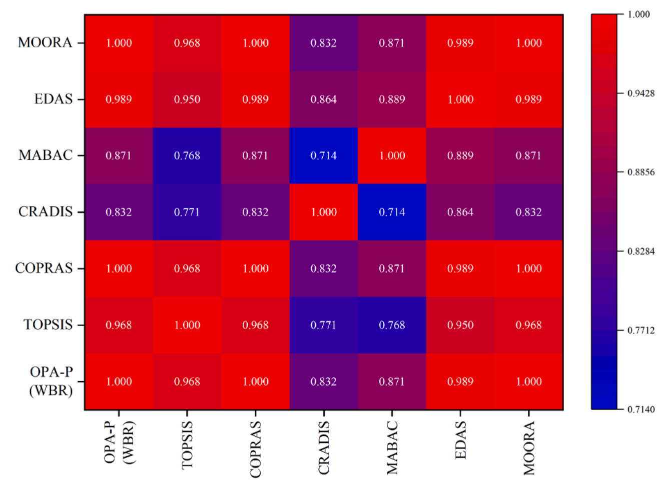


Fig. 6. Correlation heat map of alternative rankings of multiple MADM methods.

Table 5 presents the weight-based alternative ranking (WBR) of OPA-P alongside rankings from other MADM comparison methods. Fig. 6 shows the heat map of Spearman correlation coefficients. The results indicate a positive correlation between the WBR of OPA-P and the rankings of most benchmark methods, with Spearman coefficients above 0.832. Despite minor ranking differences across methods, A8 and A3 are consistently ranked as the top two, in line with the adversarial Hasse diagram results based on NDA in OPA-P. The specific causes of these discrepancies are not analyzed, as the benchmark methods rely on different assumptions and axioms, making the causes context-dependent. A more significant factor is the performance of the methods when the data is perturbed, which will be discussed next.

Table 6 presents the ranking results based on the probability of each alternative appearing in various rankings after 500 iterations. The CR indicator shows that the WBR of OPA-P and COPRAS rankings are similar, both converging probabilistically to the original expert rankings. The CR value for NDA and NDD in OPA-P is 1, indicating a shift in one alternative between adjacent ranks, specifically, A2 moved from third to fourth. In contrast, the CR values for other methods exceed 10, suggesting significant ranking reversals and poorer performance. The CP indicator reveals that NDA of OPA-P performs best, indicating a more concentrated probability distribution and higher reliability. Compared to NDA and WBR of OPA-P, NDD of OPA-P has a CP value of 2.2017, in-

Table 5
Alternative ranking results of multiple MADM methods.

	OPA-P (WBR)	TOPSIS	COPRAS	CRADIS	MABAC	EDAS	MOORA
A1	7	6	7	5	8	7	7
A2	8	7	8	9	11	9	8
A3	2	2	2	1	2	2	2
A4	12	12	12	13	6	11	12
A5	4	4	4	2	5	3	4
A6	11	11	11	14	10	12	11
A7	3	3	3	8	4	4	3
A8	1	1	1	6	1	1	1
A9	13	14	13	12	12	13	13
A10	10	8	10	10	13	10	10
A11	6	9	6	3	3	6	6
A12	9	10	9	7	9	8	9
A13	5	5	5	4	7	5	5
A14	14	13	14	11	14	14	14
A15	15	15	15	15	15	15	15

Table 6
Results for expert ranking perturbation.

	OPA-P			TOPSIS	COPRAS	CRADIS	MABAC	EDAS	MOORA
	WBR	NDA	NDD						
A1	7	3	3	6	7	5	8	7	7
A2	8	4	4	2	8	9	11	9	8
A3	2	1	1	12	2	1	2	2	2
A4	12	4	4	3	12	2	6	11	11
A5	4	2	2	11	4	13	7	12	12
A6	11	4	4	4	11	14	5	3	4
A7	3	2	5	1	3	8	10	4	3
A8	1	1	2	13	1	6	4	1	1
A9	13	5	5	14	13	12	1	13	13
A10	10	3	3	7	10	10	12	10	10
A11	6	3	3	8	6	3	13	6	6
A12	9	4	4	9	9	7	3	8	9
A13	5	3	3	10	5	4	9	5	5
A14	14	4	4	5	14	11	14	14	14
A15	15	5	5	15	15	15	15	15	15
CR	0.0000	1.0000	1.0000	23.3238	0.0000	15.5563	18.3303	12.7279	10.6771
CP	2.6676	2.8892	2.2017	2.4195	2.6676	2.7247	2.4731	2.7108	2.6174

dicating poorer performance and suggesting that level extraction based on the NDD rule is less effective than that based on the NDA rule in this scenario. In summary, the proposed OPA-P method outperforms others in both the convergence of final rankings after expert ranking perturbations and the reliability of occurrence probabilities. Additionally, it is important to note that benchmark methods rely on decision matrices as input and require prior determination of attribute weights, further emphasizing the advantages of the proposed method in simultaneously determining weights for experts, attributes, and alternatives, as well as identifying Pareto-optimal solutions.

6. Concluding remarks

This study introduces OPA-P, a novel approach designed to address complex decision-making challenges in IESS, particularly in situations involving information uncertainty, multi-stakeholder involvement, and Pareto-optimal identification. Specifically, OPA-P establishes a decision weight optimization model that simultaneously determines the weights of experts, attributes, and alternatives based on more reliable and accessible ordinal rankings obtained from experts as input data. This is achieved without requiring data standardization or expert opinion aggregation, making the process more efficient and accessible. The core innovation of OPA-P lies in the integration of a partial-order cumulative transformation technique with a solid theoretical foundation, which improves the stability of decision-making processes and aids in identifying potential Pareto-optimal solutions. Additionally, the algorithm of generating the adversarial Hasse diagram significantly improves the identification of both Pareto-optimal and suboptimal alternatives, thereby simplifying alternative dominance structures. The practical implications of OPA-P in disaster management are notable, as it offers a structured approach for addressing the complexities and uncertainties commonly encountered in disaster response. When applied to real-world cases, such as the Zhengzhou mega-rainstorm disaster, OPA-P shows potential for improving decision-making processes and supporting more effective response outcomes in emergency scenarios. This approach provides a useful tool for decision-makers in disaster management by facilitating more informed and systematic decisions in the face of uncertainty.

It is important to note that the conclusions and findings of this study are based on a limited set of case scenarios, necessitating further exploration of the practical applications of OPA-P to confirm its effectiveness. Additionally, the implementation of IESS using OPA-P assumes attribute independence, which may not fully reflect the complexities of real-world situations. Future research should refine OPA-P by considering attribute interdependencies, thereby enhancing its applicability in dynamic and complex disaster management contexts. Moreover, this study does not account for the influence of DMs' risk preferences on the decision-making outcomes. Future studies could incorporate these preferences into the OPA-P model, improving its adaptability in real-world decision-making contexts.

CRediT authorship contribution statement

Renlong Wang: Writing - original draft, Conceptualization, Methodology, Validation, Software; **Rui Shen:** Data curation, Validation, Visu-

alization, Formal analysis; **Shutian Cui:** Validation, Visualization, Formal analysis; **Xueyan Shao:** Writing - review & editing, Validation, Investigation; **Hong Chi:** Supervision, Investigation; **Mingang Gao:** Supervision, Writing - review & editing, Funding acquisition, Investigation.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Technical proof

Proof of Theorem 1

Proof. Given $(\mathcal{X}, \preceq_1) \subseteq (\mathcal{Y}, \preceq_2)$, by Definition 5, we have $\mathcal{X}^-_{x,1} \subseteq \mathcal{Y}^-_{x,2}$ and $\mathcal{X}^-_{y,1} \subseteq \mathcal{Y}^-_{y,2}$ for all $x, y \in \mathcal{X}$. By Lemma 1, $x \preceq_1 y$ implies that $x \in \mathcal{X}^-_y$. Since $\mathcal{X}^-_{y,1} \subseteq \mathcal{Y}^-_{y,2}$, then $x \in \mathcal{Y}^-_{y,2}$ such that $x \preceq_2 y$. □ □

Proof of Theorem 2

Proof. Let $\mathcal{I}^{-AC}_{i_1} = \{i_2 \in \mathcal{I} : i_2 \preceq_{AC} i_1\}$ and $\mathcal{I}^{-POCT}_{i_1} = \{i_2 \in \mathcal{I} : i_2 \preceq_{POCT} i_1\}$ denote the lower set of $i_1 \in \mathcal{I}$ on $(\mathcal{I}, \preceq_{AC})$ and $(\mathcal{I}, \preceq_{POCT})$, respectively. For $i_2 \in \mathcal{I}^{-AC}_{i_1}$, there exists $i_2 \preceq_{AC} i_1$, which yields $w^{\mathcal{I}\mathcal{J}}_{i_2 j} \leq w^{\mathcal{I}\mathcal{J}}_{i_1 j}$ for all $j \in \mathcal{J}$. It follows that $\sum_{j=1}^{l} w^{\mathcal{I}\mathcal{J}}_{i_2 j} \leq \sum_{j=1}^{l} w^{\mathcal{I}\mathcal{J}}_{i_1 j}$ for $l = 1, \ldots, J$ such that $i_2 \in \mathcal{I}^{-POCT}_{i_1}$, which implies that $\mathcal{I}^{-AC}_{i_1} \subseteq \mathcal{I}^{-POCT}_{i_1}$. By Definition 5, we have $(\mathcal{I}, \preceq_{AC}) \subseteq (\mathcal{I}, \preceq_{POCT})$. □ □

Proof of Theorem 3

Proof. Prove by mathematical induction. Consider $\mathbf{W}^{\mathcal{I}\mathcal{J}}$, and its elements $w^{\mathcal{I}\mathcal{J}}_{ij}$ can be decomposed into $w^{\mathcal{J}}_j \times v^{\mathcal{I}\mathcal{J}}_{ij}$, where $w^{\mathcal{J}}_j$ signifies the attribute weight computed in OPA-P, and $v^{\mathcal{I}\mathcal{J}}_{ij}$ can be interpreted as an unweighted utility of alternative $i \in \mathcal{I}$ with respect to the attribtue $j \in \mathcal{J}$. Given $i_1 \preceq_{AC} i_2$, it follows that $w^{\mathcal{I}\mathcal{J}}_{i_1 j} \leq w^{\mathcal{I}\mathcal{J}}_{i_2 j} \Leftrightarrow v^{\mathcal{I}\mathcal{J}}_{i_1 j} \leq v^{\mathcal{I}\mathcal{J}}_{i_2 j}$ such that:

$$\begin{aligned} &(w^{\mathcal{I}\mathcal{J}}_{i_2 1} - w^{\mathcal{I}\mathcal{J}}_{i_1 1}) + (w^{\mathcal{I}\mathcal{J}}_{i_2 2} - w^{\mathcal{I}\mathcal{J}}_{i_1 2}) + \cdots + (w^{\mathcal{I}\mathcal{J}}_{i_2 J} - w^{\mathcal{I}\mathcal{J}}_{i_1 J}) \geq 0 \\ \Leftrightarrow &w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 2} - v^{\mathcal{I}\mathcal{J}}_{i_1 2}) + \cdots + w^{\mathcal{J}}_J (v^{\mathcal{I}\mathcal{J}}_{i_2 J} - v^{\mathcal{I}\mathcal{J}}_{i_1 J}) \geq 0. \end{aligned} \tag{A.1}$$

When $r = 2$, there exists $w^{\mathcal{J}}_1 \geq w^{\mathcal{J}}_2$ and $v^{\mathcal{I}\mathcal{J}}_{i_2 1} \geq v^{\mathcal{I}\mathcal{J}}_{i_1 1}$ such that:

$$w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) \geq w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}). \tag{A.2}$$

It follows that:

$$\begin{aligned} &w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 2} - v^{\mathcal{I}\mathcal{J}}_{i_1 2}) \\ \geq &w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 2} - v^{\mathcal{I}\mathcal{J}}_{i_1 2}) \\ = &w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1} + v^{\mathcal{I}\mathcal{J}}_{i_2 2} - v^{\mathcal{I}\mathcal{J}}_{i_1 2}) \geq 0. \end{aligned} \tag{A.3}$$

When $r = l$, there exists $w^{\mathcal{J}}_{l-1} \geq w^{\mathcal{J}}_l$ and $v^{\mathcal{I}\mathcal{J}}_{i_2 j} \geq v^{\mathcal{I}\mathcal{J}}_{i_1 j}$ for all $j \in [l]$ such that:

$$\begin{aligned} &w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + \cdots + w^{\mathcal{J}}_l (v^{\mathcal{I}\mathcal{J}}_{i_2 l} - v^{\mathcal{I}\mathcal{J}}_{i_1 l}) \\ \geq &w^{\mathcal{J}}_l (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1} + \cdots + v^{\mathcal{I}\mathcal{J}}_{i_2 l} - v^{\mathcal{I}\mathcal{J}}_{i_1 l}) \geq 0. \end{aligned} \tag{A.4}$$

Thus, when $r = J$, there exists:

$$\begin{aligned} &w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + \cdots + w^{\mathcal{J}}_J (v^{\mathcal{I}\mathcal{J}}_{i_2 J} - v^{\mathcal{I}\mathcal{J}}_{i_1 J}) \\ \geq &w^{\mathcal{J}}_J (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1} + \cdots + v^{\mathcal{I}\mathcal{J}}_{i_2 J} - v^{\mathcal{I}\mathcal{J}}_{i_1 J}). \end{aligned} \tag{A.5}$$

By the given premise $w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + w^{\mathcal{J}}_2 (v^{\mathcal{I}\mathcal{J}}_{i_2 2} - v^{\mathcal{I}\mathcal{J}}_{i_1 2}) + \cdots + w^{\mathcal{J}}_J (v^{\mathcal{I}\mathcal{J}}_{i_2 J} - v^{\mathcal{I}\mathcal{J}}_{i_1 J}) \geq 0$, it follows that

$$\begin{aligned} &w^{\mathcal{J}}_1 (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1}) + \cdots + w^{\mathcal{J}}_J (v^{\mathcal{I}\mathcal{J}}_{i_2 J} - v^{\mathcal{I}\mathcal{J}}_{i_1 J}) \\ \geq &w^{\mathcal{J}}_J (v^{\mathcal{I}\mathcal{J}}_{i_2 1} - v^{\mathcal{I}\mathcal{J}}_{i_1 1} + \cdots + v^{\mathcal{I}\mathcal{J}}_{i_2 J} - v^{\mathcal{I}\mathcal{J}}_{i_1 J}) \geq 0. \end{aligned} \tag{A.6}$$

Thus, $w^{POCT}_{i_1 j} \leq w^{POCT}_{i_2 j}$ holds. □ □

Appendix B. Case data and calculation results

Table B.1
Ranking of attributes and alternatives under attributes provided by experts for the case IESS.

Expert ID	Supplier ID	SC1	SC2	SC3	SC4	SC5	SC6	SC7	SC8
E1	—	1	5	2	7	4	3	6	8
E2	—	1	2	4	8	6	3	5	7
E3	—	1	4	3	8	5	2	6	7
E4	—	2	5	1	6	7	4	3	8
E5	—	3	2	4	7	1	8	6	5
	A1	1	14	10	8	3	6	8	15
	A2	9	10	6	10	15	11	11	5
	A3	2	15	5	15	6	15	10	1
	A4	10	7	4	11	9	5	3	9
	A5	3	4	15	1	4	7	7	13
	A6	4	13	3	12	7	8	1	6
	A7	8	3	14	2	5	3	14	8
E1	A8	7	9	12	3	10	12	4	4
	A9	11	2	11	5	1	2	9	10
	A10	6	12	1	14	11	13	2	3
	A11	12	1	13	4	2	1	12	11
	A12	14	6	2	13	14	14	13	2
	A13	5	11	7	9	8	4	15	12
	A14	13	8	8	6	12	9	5	7
	A15	15	5	9	7	13	10	6	14
	A1	13	1	4	11	3	1	15	10
	A2	4	15	11	3	7	13	10	3
	A3	1	14	10	9	12	14	9	6
	A4	7	6	7	4	6	5	5	11
	A5	2	9	12	5	5	8	11	7
	A6	11	4	6	6	11	2	3	12
	A7	15	2	3	12	10	9	4	15
E2	A8	3	7	14	2	1	4	12	4
	A9	14	3	2	15	9	6	1	14
	A10	6	13	13	7	14	11	14	2
	A11	12	5	1	14	15	3	2	13
	A12	5	12	15	1	2	12	13	8
	A13	10	8	5	13	8	7	8	5
	A14	8	10	9	8	4	10	6	9
	A15	9	11	8	10	13	15	7	1
	A1	7	9	14	1	7	13	15	6
	A2	15	6	4	11	12	12	2	5
	A3	2	14	15	2	2	3	10	12
	A4	14	2	1	15	4	2	14	14
	A5	1	13	13	7	3	14	9	1
	A6	13	5	3	12	5	1	1	15
	A7	6	10	12	6	11	10	12	4
E3	A8	3	15	11	3	1	4	11	13
	A9	12	1	2	14	10	9	3	7
	A10	5	11	10	8	13	11	13	3
	A11	10	3	5	4	9	5	4	10
	A12	8	7	8	13	15	15	8	2
	A13	11	4	6	5	6	6	5	8
	A14	4	12	9	9	14	8	7	11
	A15	9	8	7	10	8	7	6	9
	A1	11	6	8	14	15	14	5	4
	A2	6	10	7	13	9	6	4	10
	A3	15	5	12	8	4	7	10	7
	A4	7	9	6	12	14	5	3	11
	A5	10	7	3	5	3	13	2	3
	A6	3	4	13	4	13	1	15	15
	A7	12	1	9	9	2	2	9	14
E4	A8	9	11	4	15	8	8	6	6
	A9	5	8	11	6	10	12	11	5
	A10	2	14	5	7	6	9	7	8
	A11	13	3	1	10	5	3	1	12
	A12	8	12	10	1	11	10	14	2
	A13	14	2	2	11	1	4	8	13
	A14	1	13	15	2	12	15	13	1
	A15	4	15	14	3	7	11	12	9
	A1	4	14	15	3	10	15	6	4
	A2	14	7	1	14	4	10	11	5
	A3	10	1	2	12	9	2	1	15
	A4	3	12	11	1	15	14	12	6
	A5	13	3	6	10	8	1	5	14
	A6	9	8	10	11	14	4	15	9
	A7	15	2	3	5	2	3	4	10
E5	A8	2	13	12	2	1	9	10	1
	A9	6	11	7	13	11	13	7	7
	A10	8	9	8	15	5	5	8	13
	A11	1	15	13	4	12	12	13	2
	A12	11	4	4	9	6	6	2	12
	A13	7	5	9	6	3	7	3	11
	A14	12	6	5	7	7	8	14	8
	A15	5	10	14	8	13	11	9	3

Table B.2
Optimal weights for the case IESS.

Expert ID	Supplier ID	C1	C2	C3	C4	C5	C6	C7	C8
E1	A1	0.0119	0.0001	0.0009	0.0004	0.0016	0.0012	0.0004	0.0000
	A2	0.0021	0.0004	0.0019	0.0003	0.0001	0.0005	0.0002	0.0006
	A3	0.0083	0.0000	0.0022	0.0000	0.0009	0.0001	0.0003	0.0015
	A4	0.0018	0.0006	0.0027	0.0002	0.0005	0.0015	0.0011	0.0003
	A5	0.0065	0.0011	0.0001	0.0017	0.0013	0.0010	0.0005	0.0001
	A6	0.0053	0.0002	0.0033	0.0002	0.0008	0.0009	0.0020	0.0005
	A7	0.0026	0.0013	0.0002	0.0012	0.0011	0.0022	0.0001	0.0003
	A8	0.0031	0.0004	0.0005	0.0009	0.0004	0.0004	0.0009	0.0007
	A9	0.0014	0.0017	0.0007	0.0006	0.0030	0.0028	0.0004	0.0002
	A10	0.0037	0.0002	0.0059	0.0001	0.0003	0.0003	0.0014	0.0008
	A11	0.0011	0.0024	0.0004	0.0008	0.0021	0.0040	0.0002	0.0002
	A12	0.0005	0.0007	0.0042	0.0001	0.0001	0.0002	0.0001	0.0010
	A13	0.0044	0.0003	0.0016	0.0003	0.0006	0.0018	0.0000	0.0001
	A14	0.0008	0.0005	0.0013	0.0005	0.0003	0.0007	0.0007	0.0004
	A15	0.0002	0.0009	0.0011	0.0004	0.0002	0.0006	0.0006	0.0001
E2	A1	0.0012	0.0089	0.0020	0.0003	0.0016	0.0059	0.0001	0.0004
	A2	0.0080	0.0002	0.0005	0.0012	0.0008	0.0004	0.0005	0.0014
	A3	0.0178	0.0004	0.0007	0.0004	0.0003	0.0002	0.0006	0.0008
	A4	0.0047	0.0028	0.0012	0.0010	0.0009	0.0022	0.0013	0.0003
	A5	0.0125	0.0016	0.0004	0.0008	0.0011	0.0013	0.0004	0.0007
	A6	0.0021	0.0040	0.0014	0.0007	0.0003	0.0042	0.0020	0.0002
	A7	0.0004	0.0062	0.0024	0.0002	0.0004	0.0011	0.0016	0.0001
	A8	0.0098	0.0023	0.0002	0.0016	0.0030	0.0027	0.0003	0.0011
	A9	0.0007	0.0049	0.0031	0.0000	0.0005	0.0019	0.0036	0.0001
	A10	0.0056	0.0006	0.0003	0.0006	0.0001	0.0007	0.0001	0.0018
	A11	0.0016	0.0033	0.0045	0.0001	0.0001	0.0033	0.0025	0.0002
	A12	0.0066	0.0008	0.0001	0.0022	0.0021	0.0005	0.0002	0.0006
	A13	0.0026	0.0019	0.0017	0.0001	0.0006	0.0016	0.0008	0.0009
	A14	0.0039	0.0013	0.0008	0.0005	0.0013	0.0009	0.0011	0.0005
	A15	0.0032	0.0010	0.0010	0.0003	0.0002	0.0001	0.0009	0.0025
E3	A1	0.0023	0.0004	0.0001	0.0011	0.0005	0.0003	0.0000	0.0004
	A2	0.0002	0.0007	0.0013	0.0001	0.0002	0.0004	0.0010	0.0005
	A3	0.0062	0.0001	0.0001	0.0008	0.0012	0.0024	0.0002	0.0001
	A4	0.0004	0.0016	0.0030	0.0000	0.0008	0.0031	0.0001	0.0001
	A5	0.0089	0.0001	0.0002	0.0003	0.0010	0.0002	0.0003	0.0013
	A6	0.0006	0.0008	0.0016	0.0001	0.0007	0.0045	0.0015	0.0000
	A7	0.0028	0.0003	0.0003	0.0003	0.0002	0.0007	0.0001	0.0006
	A8	0.0049	0.0000	0.0003	0.0006	0.0018	0.0020	0.0002	0.0001
	A9	0.0008	0.0022	0.0021	0.0000	0.0003	0.0008	0.0008	0.0003
	A10	0.0033	0.0003	0.0004	0.0002	0.0001	0.0005	0.0001	0.0007
	A11	0.0013	0.0012	0.0011	0.0005	0.0003	0.0017	0.0007	0.0002
	A12	0.0019	0.0006	0.0006	0.0001	0.0000	0.0001	0.0003	0.0009
	A13	0.0010	0.0010	0.0009	0.0004	0.0006	0.0014	0.0006	0.0003
	A14	0.0040	0.0002	0.0005	0.0002	0.0001	0.0010	0.0004	0.0001
	A15	0.0016	0.0005	0.0008	0.0002	0.0004	0.0012	0.0005	0.0002
E4	A1	0.0004	0.0004	0.0016	0.0000	0.0000	0.0001	0.0009	0.0004
	A2	0.0011	0.0002	0.0019	0.0001	0.0002	0.0006	0.0011	0.0001
	A3	0.0001	0.0005	0.0006	0.0003	0.0005	0.0005	0.0004	0.0002
	A4	0.0009	0.0003	0.0022	0.0001	0.0000	0.0007	0.0013	0.0001
	A5	0.0005	0.0004	0.0039	0.0004	0.0006	0.0001	0.0017	0.0005
	A6	0.0020	0.0006	0.0005	0.0005	0.0001	0.0018	0.0000	0.0000
	A7	0.0003	0.0014	0.0013	0.0002	0.0007	0.0012	0.0004	0.0000
	A8	0.0006	0.0002	0.0032	0.0000	0.0002	0.0004	0.0007	0.0003
	A9	0.0013	0.0003	0.0008	0.0004	0.0002	0.0002	0.0003	0.0003
	A10	0.0025	0.0001	0.0027	0.0003	0.0003	0.0003	0.0006	0.0002
	A11	0.0002	0.0008	0.0071	0.0002	0.0004	0.0010	0.0024	0.0001
	A12	0.0008	0.0001	0.0011	0.0012	0.0001	0.0003	0.0001	0.0006
	A13	0.0001	0.0010	0.0050	0.0001	0.0010	0.0008	0.0005	0.0001
	A14	0.0036	0.0001	0.0001	0.0008	0.0001	0.0000	0.0002	0.0009
	A15	0.0016	0.0000	0.0003	0.0007	0.0003	0.0002	0.0002	0.0002
E5	A1	0.0053	0.0007	0.0002	0.0028	0.0053	0.0001	0.0019	0.0032
	A2	0.0005	0.0047	0.0089	0.0002	0.0160	0.0007	0.0007	0.0027
	A3	0.0018	0.0178	0.0062	0.0005	0.0064	0.0031	0.0059	0.0001
	A4	0.0065	0.0016	0.0010	0.0051	0.0007	0.0002	0.0005	0.0022
	A5	0.0008	0.0098	0.0028	0.0008	0.0078	0.0045	0.0022	0.0003
	A6	0.0021	0.0039	0.0013	0.0006	0.0015	0.0020	0.0001	0.0013
	A7	0.0002	0.0125	0.0049	0.0019	0.0249	0.0024	0.0027	0.0011
	A8	0.0083	0.0012	0.0008	0.0036	0.0356	0.0008	0.0009	0.0071
	A9	0.0037	0.0021	0.0023	0.0003	0.0042	0.0003	0.0016	0.0019
	A10	0.0026	0.0032	0.0019	0.0001	0.0133	0.0017	0.0013	0.0005
	A11	0.0119	0.0004	0.0006	0.0023	0.0032	0.0004	0.0004	0.0050
	A12	0.0014	0.0080	0.0040	0.0009	0.0111	0.0014	0.0042	0.0006
	A13	0.0031	0.0066	0.0016	0.0016	0.0195	0.0012	0.0033	0.0008
	A14	0.0011	0.0056	0.0033	0.0013	0.0093	0.0010	0.0002	0.0016
	A15	0.0044	0.0026	0.0004	0.0011	0.0023	0.0005	0.0011	0.0039

Table B.3
Partial-order cumulative transformation weights of the case IESS.

	$w_{j_1}^{POCT}$	$w_{j_2}^{POCT}$	$w_{j_3}^{POCT}$	$w_{j_4}^{POCT}$	$w_{j_5}^{POCT}$	$w_{j_6}^{POCT}$	$w_{j_7}^{POCT}$	$w_{j_8}^{POCT}$
A1	0.0211	0.0301	0.0407	0.0454	0.0531	0.0563	0.0607	0.0653
A2	0.0119	0.0290	0.0351	0.0496	0.0521	0.0556	0.0608	0.0627
A3	0.0342	0.0435	0.0624	0.0722	0.0785	0.0860	0.0887	0.0907
A4	0.0142	0.0173	0.0241	0.0341	0.0418	0.0461	0.0490	0.0555
A5	0.0292	0.0409	0.0539	0.0613	0.0684	0.0735	0.0763	0.0803
A6	0.0121	0.0154	0.0249	0.0330	0.0462	0.0518	0.0538	0.0559
A7	0.0063	0.0337	0.0554	0.0645	0.0721	0.0770	0.0790	0.0829
A8	0.0267	0.0678	0.0719	0.0770	0.0832	0.0862	0.0954	0.1021
A9	0.0080	0.0161	0.0272	0.0363	0.0422	0.0487	0.0516	0.0530
A10	0.0177	0.0318	0.0362	0.0474	0.0509	0.0545	0.0584	0.0597
A11	0.0161	0.0221	0.0302	0.0438	0.0541	0.0602	0.0658	0.0696
A12	0.0112	0.0247	0.0350	0.0449	0.0473	0.0523	0.0560	0.0605
A13	0.0114	0.0338	0.0446	0.0553	0.0620	0.0672	0.0694	0.0720
A14	0.0133	0.0244	0.0321	0.0382	0.0417	0.0444	0.0478	0.0512
A15	0.0111	0.0144	0.0195	0.0230	0.0256	0.0289	0.0358	0.0385

Table B.4
Descriptive statistics of the computed weight outcomes for sensitivity analysis.

	Mean	Max	Min	Standard Deviation	Variance	Skewness	Kurtosis	Coefficient of Variation
E1	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E2	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E3	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E4	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E5	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
C1	0.2821	0.3263	0.2202	0.0341	0.0012	-0.3725	-1.3224	0.1209
C2	0.1214	0.1488	0.0980	0.0164	0.0003	0.2634	-1.4249	0.1353
C3	0.1717	0.2375	0.1307	0.0331	0.0011	0.8466	-0.6099	0.1927
C4	0.0517	0.0551	0.0490	0.0018	0.0000	0.4307	-0.9298	0.0348
C5	0.1295	0.2033	0.0895	0.0383	0.0015	1.0689	-0.3561	0.2956
C6	0.1135	0.1394	0.0877	0.0144	0.0002	0.0569	-0.8765	0.1267
C7	0.0760	0.0909	0.0680	0.0076	0.0001	1.0011	-0.4367	0.0998
C8	0.0541	0.0605	0.0501	0.0032	0.0000	0.8783	-0.5356	0.0599
A1	0.0683	0.0856	0.0559	0.0093	0.0001	0.4050	-1.2665	0.1362
A2	0.0555	0.0639	0.0485	0.0045	0.0000	0.2496	-1.0751	0.0813
A3	0.0841	0.0929	0.0686	0.0080	0.0001	-1.0532	-0.3691	0.0952
A4	0.0623	0.0696	0.0545	0.0042	0.0000	-0.1595	-0.7932	0.0672
A5	0.0881	0.0967	0.0798	0.0047	0.0000	0.1111	-0.7965	0.0536
A6	0.0676	0.0779	0.0540	0.0070	0.0000	-0.5076	-0.8752	0.1035
A7	0.0694	0.0842	0.0598	0.0076	0.0001	0.9465	-0.4578	0.1089
A8	0.0868	0.1039	0.0715	0.0091	0.0001	0.2538	-0.9885	0.1046
A9	0.0577	0.0647	0.0494	0.0047	0.0000	-0.2852	-1.3624	0.0807
A10	0.0639	0.0734	0.0554	0.0053	0.0000	0.2339	-1.3156	0.0826
A11	0.0806	0.0992	0.0687	0.0095	0.0001	0.9569	-0.4535	0.1178
A12	0.0541	0.0607	0.0488	0.0034	0.0000	0.4575	-1.0154	0.0636
A13	0.0702	0.0816	0.0602	0.0062	0.0000	0.2458	-1.2380	0.0876
A14	0.0524	0.0581	0.0461	0.0033	0.0000	-0.2151	-1.0294	0.0638
A15	0.0391	0.0428	0.0351	0.0021	0.0000	-0.2569	-0.8675	0.0545

Table B.5
Decision data for the comparison MADM methods.

v_{ij}^{IJ}	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.0863	0.0709	0.0361	0.0899	0.0449	0.0816	0.0476	0.0730
A2	0.0487	0.0419	0.1091	0.0371	0.0862	0.0279	0.0504	0.0880
A3	0.1399	0.1269	0.0737	0.0391	0.0463	0.0677	0.1067	0.0448
A4	0.0585	0.0466	0.0760	0.1251	0.0145	0.0827	0.0620	0.0498
A5	0.1195	0.0878	0.0557	0.0782	0.0588	0.0763	0.0735	0.0481
A6	0.0495	0.0641	0.0609	0.0410	0.0169	0.1439	0.0807	0.0332
A7	0.0258	0.1465	0.0685	0.0743	0.1361	0.0816	0.0706	0.0349
A8	0.1092	0.0277	0.0376	0.1309	0.2043	0.0677	0.0432	0.1544
A9	0.0323	0.0756	0.0677	0.0254	0.0409	0.0644	0.0966	0.0465
A10	0.0724	0.0297	0.0842	0.0254	0.0703	0.0376	0.0504	0.0664
A11	0.0659	0.0547	0.1031	0.0762	0.0304	0.1117	0.0894	0.0946
A12	0.0458	0.0689	0.0752	0.0879	0.0668	0.0269	0.0706	0.0614
A13	0.0458	0.0729	0.0812	0.0489	0.1111	0.0730	0.0749	0.0365
A14	0.0548	0.0520	0.0451	0.0645	0.0553	0.0387	0.0375	0.0581
A15	0.0450	0.0338	0.0271	0.0528	0.0169	0.0279	0.0476	0.1145
w_j^{J}	0.2444	0.1481	0.1329	0.0512	0.2007	0.0931	0.0694	0.0602

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